SICE Annual Conference 2010
August 18-21, 2010, The Grand Hotel, Taipei, Taiwan

Modelling and Control of Hyper-Redundancy Mobile Manipulator Bracing Multi-Elbows for High Accuracy / Low-Energy Consumption

Geng Wang[1] and Mamoru Minami[2]

[1]Graduate School of Natural Science and Technology, Okayama University
Tsushimanaka3-1-1 , Okayama, JAPAN
(Tel/Fax: +81-86-251-8233; Email: owg_cyo@suri.sys.okayama-u.ac.jp)
[2]Graduate School of Natural Science and Technology, Okayama University
Tsushimanaka3-1-1 , Okayama, JAPAN
(Tel/Fax: +81-86-251-8233; E-mail: minami@sys.okayama-u.ac.jp)

Abstract— **This paper presents a practical dynamical model of hyper-redundant mobile manipulator whose plural intermediate links are being braced with environment. To overcome the conflict between the required high-redundancy for dexterous manipulation and heavy weight stemming from the high redundancy, we discuss a realizability whether the contacting and bracing motion of intermediate links with environment may simultaneously prevent from overturning and reduce energy-consumption and raise hand's trajectory tracking accuracy, inspired by human's handwriting motion with the elbow or wrist contacting to a table. Finally the simulation result shows that less-energy consumption and high trajectory tracking accuracy are achieved by conventional PD controller, compared with non-contacting and non-basement-moving manipulator condition.**

I. INTRODUCTION

Hindrances interfering realistic and practical utilization of hyper-redundant manipulator is thought to be the facts that the higher redundant degrees make the weight of the structure heavier, resulting in some difficulties in the controlling, accuracy and stability including whether the hyper-redundant mobile manipulator will overturn or not. For solving this problem we obtain some inspirations about effective motion control strategies by observing human's handwriting motion. Writing a character on a paper with contacting one's elbow as shown Fig. 1(a), which is one of the examples of human's skillful behavior thought to be exploiting the contact constraint of the elbow with the table for reducing inputting energy by countering the gravity effects with reaction forces.

On the other hand, hyper-redundant manipulator had been researched intensively and those efforts had been introduced by Chirikjian and Burdick[1] more than ten years ago, where the structure of the discussed hyper-redundant manipulator could not move in 3D space but restricted in the 2D space on a surface of table. Though considerable researches have discussed how to utilize the redundancy [2]-[5], for example avoiding obstacles [6]-[9] or optimizing the configuration concerning practical criteria [10],[11], etc.. However it seems that the merits of the hyper-redundancy has not been utilized enough effectively and practically. Here we think the reason that higher redundant degree causes heavier weight of structure, then more easily its end effector falls down by gravity influence, which has the control precision of the end-effector getting worse.

Therefore up to now there are many researches to discuss the effectiveness and accuracy of the hyper-redundant manipulator with constraint due to contact with the environment. West and Asada[12] presented a general kinematics contact model for the design of hybrid position/force controllers for constrained manipulator. And then a multi-contact kinematic model to control manipulator's contact motion was also presented in[13],[14], in which they assumed the contact environment as a spring model. However actually the contact environment is almost rigid even if it can happen changed, which must be also offered a rather large force on a premise that the contact enviroment can not be broken. Moreover the contact point of manipulator will shake with respect to the contact environment due to the unstable of the contact force. So this spring contact environment model is somewhat not the best correct approach to represent contacting nature. Contrarily in this paper we will discuss a model without contacting deformation of environment.

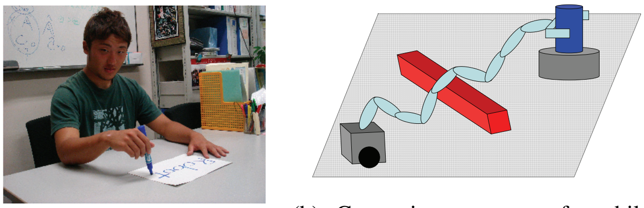
In this research, we propose a new dynamical mobile model of manipulator with multi-elbow and basement which is shown in Fig.1(b) depicting 10-links redundant manipulator whose intermediate links contact to the floor at two points, while the hand is required to draw a circle in 3D task space, comprising manipulator's dynamics and geometrical constraint conditions, realized through the synthesization of multi-constraint condition of elbows and mobile manipulator's motion equation. Even though a controller used for end-effector's trajectory tracking task is simple PD controller, it has been evaluated how the contacting strategy and improve tracking accuracy and energy-saving performances.

II. MODELLING OF HYPER-REDUNDANT MOBILE MANIPULATOR WITH CONSTRAINT

A. Manipulator's Model with Hand's Constraint

To make the explanation of constraint motion with multi-elbow be easily understandable, we discuss firstly about the model of the manipulator whose end-effector is contacing rigid environment without elasticity. Equation of motion of manipulator composing rigid structure of s links, and

PR0001/10/0000-2371 ¥400



(a) Human's writing motion (b) Contacting strategy of mobile manipulator

Fig. 1. The sketch picture of Hyper-Redundant Manipulator with elbows

also contact relation between manipulator's end-effector and definition of constraint surface should be introduced firstly. L represents lagrangian, $\boldsymbol{q} \in R^s$ represents the general coordinate, $\boldsymbol{\tau} \in R^s$ represents the general input. u is the unknown constant of lagrange, f_t is the friction. Manipulator hand's lagrange condition equation can be expressed as follows

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\boldsymbol{q}}}\right) - \left(\frac{\partial L}{\partial \boldsymbol{q}}\right) = \boldsymbol{\tau} + \left(\frac{\partial C}{\partial \boldsymbol{q}^T}\right)^T u - \left(\frac{\partial \boldsymbol{r}}{\partial \boldsymbol{q}^T}\right)^T \frac{\dot{\boldsymbol{r}}}{\|\dot{\boldsymbol{r}}\|} f_t \tag{1}$$

Here according to the kinematic relation, manipulator hand's position/posture vector $\boldsymbol{r} \in R^s$ and scalar function, a single constraint condition C which used to express the hypersurface can be expressed as

$$\boldsymbol{r} = \boldsymbol{r}(\boldsymbol{q}) \tag{2}$$

$$C(\boldsymbol{r}(\boldsymbol{q})) = 0 \tag{3}$$

The freedom of the end-effctor to move freely in the direction of non-constraint is left to be more than one, so here $s > 1$. If we set f_n to indicate the constraint force of manipulator hand, then the relation of u and f_n can be expressed as

$$u = f_n / \left\|\frac{\partial C}{\partial \boldsymbol{r}^T}\right\| \tag{4}$$

$\|\partial C/\partial \boldsymbol{r}^T\|$ shows Euclidean norm of vector $\partial C/\partial \boldsymbol{r}^T$. Then manipulator's motion equation can be derived from the combination of Eq(1) and Eq(4) with viscous friction of joints [15].

$$\begin{aligned} &\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{h}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{D}\dot{\boldsymbol{q}} \\ = \ &\boldsymbol{\tau} + \left\{\left(\frac{\partial C}{\partial \boldsymbol{q}^T}\right)^T / \left\|\frac{\partial C}{\partial \boldsymbol{r}^T}\right\|\right\} f_n - \left(\frac{\partial \boldsymbol{r}}{\partial \boldsymbol{q}^T}\right)^T \frac{\dot{\boldsymbol{r}}}{\|\dot{\boldsymbol{r}}\|} f_t \end{aligned} \tag{5}$$

$\boldsymbol{M}$ is inertia matrix of $s \times s$, $\boldsymbol{h}$ and $\boldsymbol{g}$ are $s \times 1$ vectors which indicate the effects from coriolis force, centrifugal force and gravity, $\boldsymbol{D}$ is a $s \times s$ matrix which indicates the coefficient of joints' viscous friction, expressed as $\boldsymbol{D} = diag[D_1, D_2, \cdots, D_s]$. $\boldsymbol{q}$ is the joint angle and $\boldsymbol{\tau}$ is the input torque.

B. Model with Multiple Constraints

Here we consider a motion of a manipulator having s links whose elbows are contactig at p points to the environments defined as

$$C_i(\boldsymbol{r}_i(\boldsymbol{q})) = 0, \quad (i = 1, 2, \cdots, p) \tag{6}$$

where $\boldsymbol{r}_i$ is the equation of position and posture of link i contacting with constraint, like Eq(2).

$$\boldsymbol{r}_i = \boldsymbol{r}_i(\boldsymbol{q}) \tag{7}$$

The Eq(5) describes a motion of the manipulator whose hand is constraint. Under the situation with the i-th link contacting, then we can define following two vectors concering i-th constraint condition C_i as follows,

$$\left(\frac{\partial C_i}{\partial \boldsymbol{q}^T}\right)^T / \left\|\frac{\partial C_i}{\partial \boldsymbol{r}^T}\right\| = \boldsymbol{j_c}_i^T \tag{8}$$

$$\left(\frac{\partial \boldsymbol{r}_i}{\partial \boldsymbol{q}^T}\right)^T \frac{\dot{\boldsymbol{r}}_i}{\|\dot{\boldsymbol{r}}_i\|} = \boldsymbol{j_t}_i^T \tag{9}$$

Accumulating all the above vectors ($i = 1, 2, \cdots, p$) when p links are contacting, the next is redefined.

$$\boldsymbol{J_c}^T = [\boldsymbol{j_c}_1^T, \ \boldsymbol{j_c}_2^T, \ \cdots, \ \boldsymbol{j_c}_p^T] \tag{10}$$

$$\boldsymbol{J_t}^T = [\boldsymbol{j_t}_1^T, \ \boldsymbol{j_t}_2^T, \ \cdots, \ \boldsymbol{j_t}_p^T] \tag{11}$$

$$\boldsymbol{f_n} = [f_{n1}, \ f_{n2}, \ \cdots, \ f_{np}]^T \tag{12}$$

$$\boldsymbol{f_t} = [f_{t1}, \ f_{t2}, \ \cdots, \ f_{tp}]^T \tag{13}$$

$\boldsymbol{J_c}^T, \boldsymbol{J_t}^T$ are $s \times p$ matrix, $\boldsymbol{f_n}, \boldsymbol{f_t}$ are $p \times 1$ vectors. Considering about p constraints of the intermediate links, the manipulator's equation of motion can be expressed as

$$\begin{aligned} &\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{h}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{D}\dot{\boldsymbol{q}} \\ = \ &\boldsymbol{\tau} + \sum_{i=1}^{p}(\boldsymbol{j_c}_i^T f_{ni}) - \sum_{i=1}^{p}(\boldsymbol{j_t}_i^T f_{ti}) \\ = \ &\boldsymbol{\tau} + \boldsymbol{J_c}^T \boldsymbol{f_n} - \boldsymbol{J_t}^T \boldsymbol{f_t} \end{aligned} \tag{14}$$

Moreover, Eq (6) is differentiated by time t two times, then we can derive the constraint condition of $\ddot{q}$.

$$\left[\frac{\partial}{\partial \boldsymbol{q}}\left(\frac{\partial C_i}{\partial \boldsymbol{q}^T}\right)\dot{\boldsymbol{q}}\right]\dot{\boldsymbol{q}} + \left(\frac{\partial C_i}{\partial \boldsymbol{q}^T}\right)\ddot{\boldsymbol{q}} = 0 \tag{15}$$

To make sure that manipulator hand is contacting with the constraint surface all the time, value of $\boldsymbol{q}(t)$ in Eq(14) has always to satisfy Eq(6) which has no relation with time t, if value of $\ddot{\boldsymbol{q}}$ in Eq(15) should have the same value with $\ddot{\boldsymbol{q}}$ in Eq(14), then value of $\boldsymbol{q}(t)$ in Eq(14) and Eq(6) always keep the same regardless of time.

C. Robot's Dynamics Including Motors

In this research, we want to evaluate the effects to increase the trajectory tracking accuracy and reduce the energy consumption used for countering the gravity force and other effects by bracing the intermediate links. Even though there is no robot's motion –robot is stopping– the energy is kept to be consuming since motors of joints have to generate torques to maintain the required robot's configuration against gravity influences. When the robot is in motion, other effects of dynamics will be added more to the gravity effect. To evaluate this kind of wasted energy consumption, we included the effects of electronic circuit of servo motor into the equation of motion of the manipulator to represent explicitely that the robot consumes energy even while stopping.

Here v_i represents motor's voltage, R_i does resistance, L_i, i_i do the inductance and electric current, θ_i does the angular phase of motor, τ_{gi} does the motor ouput torque, τ_{Li} does the load torque, v_{gi} does electromotive force, I_{mi} does the inertia moment of motor, K_{Ei} does the constant of electrmotive force, K_{Ti} does the constant of torque, d_{mi} does the viscous friction's coefficient of speed reducer. The relation of those variables are shown hereunder.

$$\begin{aligned} v_i(t) &= L_i\dot{i}_i + R_i i_i(t) + v_{gi}(t) && (16)\\ v_{gi}(t) &= K_{Ei}\dot{\theta}_i(t) && (17)\\ I_{mi}\ddot{\theta}_i &= \tau_{gi}(t) - \tau_{Li}(t) - d_{mi}\dot{\theta}_i && (18)\\ \tau_g(t) &= K_{Ti} i_i(t) && (19)\end{aligned}$$

From the relation of magnetic field and the coefficients above, $K_{Ti} = K_{Ei} (= K)$ holds for motors used. Combine Eq (17) and Eq (16), and also Eq (19) and Eq (18), we derive

$$\begin{aligned} v_i &= L_i\dot{i}_i + R_i i_i + K_i\dot{\theta}_i && (20)\\ I_{mi}\ddot{\theta}_i &= K_i i_i - \tau_{Li} - d_{mi}\dot{\theta}_i && (21)\end{aligned}$$

In the situation with motor and gear whose reduction ratio is k_i are installed onto manipulator,

$$\begin{aligned} \theta_i &= k_i q_i && (22)\\ \tau_{Li} &= \frac{\tau_i}{k_i} && (23)\end{aligned}$$

Combining Eq (20) and Eq (21) into equation with $\dot{i}_i$ and τ_i, following equations are obtained

$$\begin{aligned} L_i\dot{i}_i &= v_i - R_i i_i - K_i k_i \dot{q}_i && (24)\\ \tau_i &= -I_{mi}k_i^2\ddot{q}_i + K_i k_i i_i - d_{mi}k_i^2\dot{q}_i && (25)\end{aligned}$$

Then using vector and matrix to indicate Eq (24) and (25),

$$\begin{aligned} \boldsymbol{L}\dot{\boldsymbol{i}} &= \boldsymbol{v} - \boldsymbol{R}\boldsymbol{i} - \boldsymbol{K_m}\dot{\boldsymbol{q}} && (26)\\ \boldsymbol{\tau} &= -\boldsymbol{J_m}\ddot{\boldsymbol{q}} + \boldsymbol{K_m}\boldsymbol{i} - \boldsymbol{D_m}\dot{\boldsymbol{q}} && (27)\end{aligned}$$

$$\begin{aligned} \boldsymbol{v} &= [v_1, v_2, \cdots, v_s]^T\\ \boldsymbol{i} &= [i_1, i_2, \cdots, i_s]^T\end{aligned}$$

and the definitions are shown as follow, which always have positive value.

$$\begin{aligned} \boldsymbol{L} &= diag[L_1, L_2, \cdots, L_s]\\ \boldsymbol{R} &= diag[R_1, R_2, \cdots, R_s]\\ \boldsymbol{K_m} &= diag[K_{m1}, K_{m2}, \cdots, K_{ms}]\\ \boldsymbol{J_m} &= diag[J_{m1}, J_{m2}, \cdots, J_{ms}]\\ \boldsymbol{D_m} &= diag[D_{m1}, D_{m2}, \cdots, D_{ms}]\\ K_{mi} &= K_i k_i,\ J_{mi} = I_{mi}k_i^2,\ D_{mi} = d_{mi}k_i^2\end{aligned}$$

Now substitute Eq (27) into Eq (14), we get

$$\begin{aligned} &(\boldsymbol{M}(\boldsymbol{q}) + \boldsymbol{J_m})\ddot{\boldsymbol{q}} + \boldsymbol{h}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q}) + (\boldsymbol{D} + \boldsymbol{D_m})\dot{\boldsymbol{q}}\\ &= \boldsymbol{K_m}\boldsymbol{i} + \boldsymbol{J_c}^T\boldsymbol{f_n} - \boldsymbol{J_t}^T\boldsymbol{f_t} \qquad (28)\end{aligned}$$

Similar to the same relation between Eq (14) and Eq (15), the value of $\ddot{\boldsymbol{q}}$ in Eq (28) have to be identical to the the value of $\ddot{\boldsymbol{q}}$ in Eq (15) representing constrain condition.

D. Robot/Motor Equation with Contact Constraint

To make sure that $\ddot{\boldsymbol{q}}$ in Eq (28) and (15) be identical, constraint force $\boldsymbol{f_n}$ is subordinately decided by simultaneous equation. Then Eq (28),(15) should be transformed as follow

$$\begin{aligned} &(\boldsymbol{M} + \boldsymbol{J_m})\ddot{\boldsymbol{q}} - \boldsymbol{J_c}^T\boldsymbol{f_n}\\ &\quad = \boldsymbol{K_m}\boldsymbol{i} - \boldsymbol{h} - \boldsymbol{g} - (\boldsymbol{D} + \boldsymbol{D_m})\dot{\boldsymbol{q}} - \boldsymbol{J_t}^T\boldsymbol{f_t} \qquad (29)\\ &(\frac{\partial C_i}{\partial \boldsymbol{q}^T})\ddot{\boldsymbol{q}} = -\left[\frac{\partial}{\partial \boldsymbol{q}}(\frac{\partial C_i}{\partial \boldsymbol{q}})\dot{\boldsymbol{q}}\right]\dot{\boldsymbol{q}}\\ &\qquad = -\dot{\boldsymbol{q}}^T\left[\frac{\partial}{\partial \boldsymbol{q}}(\frac{\partial C_i}{\partial \boldsymbol{q}^T})\right]\dot{\boldsymbol{q}} \qquad (30)\end{aligned}$$

Then Eqn (29),(30),(24) can be expressed as follow. Here we assumed that friction force f_{ti} is dynamic friction and define it as $f_t = 0.1 f_n (i = 1, 2, \cdots, p)$.

$$\begin{bmatrix} \boldsymbol{M}+\boldsymbol{J_m} & -\boldsymbol{j_{c1}}^T & \cdots & -\boldsymbol{j_{cp}}^T & 0 & \cdots & 0\\ \dfrac{\partial C_1}{\partial \boldsymbol{q}^T} & 0 & \cdots & 0 & 0 & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots\\ \dfrac{\partial C_p}{\partial \boldsymbol{q}^T} & 0 & \cdots & 0 & 0 & \cdots & 0\\ 0 & 0 & \cdots & 0 & L_1 & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & 0 & 0 & \cdots & L_s \end{bmatrix} \begin{bmatrix} \ddot{\boldsymbol{q}}\\ f_{n1}\\ \vdots\\ f_{np}\\ \dot{i}_1\\ \vdots\\ \dot{i}_s \end{bmatrix}$$
$$= \begin{bmatrix} \boldsymbol{K_m}\boldsymbol{i} - \boldsymbol{h} - \boldsymbol{g} - (\boldsymbol{D}+\boldsymbol{D_m})\dot{\boldsymbol{q}} - \boldsymbol{J_t}^T\boldsymbol{f_t}\\ -\dot{\boldsymbol{q}}^T\left[\dfrac{\partial}{\partial \boldsymbol{q}}(\dfrac{\partial C_1}{\partial \boldsymbol{q}^T})\right]\dot{\boldsymbol{q}}\\ \vdots\\ -\dot{\boldsymbol{q}}^T\left[\dfrac{\partial}{\partial \boldsymbol{q}}(\dfrac{\partial C_p}{\partial \boldsymbol{q}^T})\right]\dot{\boldsymbol{q}}\\ v_1 - R_1 i_1 - K_{m1}\dot{q}_1\\ \vdots\\ v_i - R_s i_s - K_{ml}\dot{q}_s \end{bmatrix} \qquad (31)$$

The inertia term $(\boldsymbol{M}+\boldsymbol{J_m})$ is $s \times s$ matrix, the coefficient vector of constraint force $\boldsymbol{j_{ci}}^T$ is $s \times 1$ vertical vector, $\partial C_i/\partial \boldsymbol{q}^T$ is $1 \times s$ horizontal vector, inductance term $\boldsymbol{L}$ is $s \times s$ diagonal matrix, therefore, the matrix of the first term in left side in Eq (31) is matrix of $(2s+p) \times (2s+p)$.

Then Eq (31) can be rewritten concisely using the definitions of Eq (10),(12),(26) as follow,

$$\begin{bmatrix} \boldsymbol{M}+\boldsymbol{J_m} & -\boldsymbol{J_c}^T & \boldsymbol{0}\\ \frac{\partial \boldsymbol{C}}{\partial \boldsymbol{q}^T} & \boldsymbol{0} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{L} \end{bmatrix} \begin{bmatrix} \ddot{\boldsymbol{q}}\\ \boldsymbol{f_n}\\ \dot{\boldsymbol{i}} \end{bmatrix}$$
$$= \begin{bmatrix} \boldsymbol{K_m}\boldsymbol{i} - \boldsymbol{h} - \boldsymbol{g} - (\boldsymbol{D}+\boldsymbol{D_m})\dot{\boldsymbol{q}} - \boldsymbol{J_t}^T\boldsymbol{f_t}\\ -\dot{\boldsymbol{q}}^T\left[\frac{\partial}{\partial \boldsymbol{q}}(\frac{\partial \boldsymbol{C}}{\partial \boldsymbol{q}^T})\right]\dot{\boldsymbol{q}}\\ \boldsymbol{v} - \boldsymbol{R}\boldsymbol{i} - \boldsymbol{K_m}\dot{\boldsymbol{q}} \end{bmatrix} \qquad (32)$$

where, $\boldsymbol{C}$ is a vector of $\boldsymbol{C} = [C_1, C_2, \cdots, C_p]^T$. Furthermore by redefining as

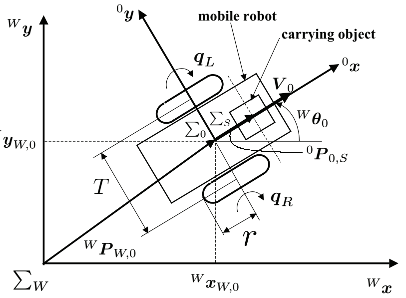


Fig. 2. Power-Wheeled-Steeling(PWS) mobile robot

$$\boldsymbol{M}^* = \begin{bmatrix} \boldsymbol{M}+\boldsymbol{J_m} & -\boldsymbol{J_c}^T & \boldsymbol{0} \\ \frac{\partial \boldsymbol{C}}{\partial \boldsymbol{q}^T} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{L} \end{bmatrix} \tag{33}$$

$$\boldsymbol{b} = \begin{bmatrix} \boldsymbol{K_m i} - \boldsymbol{h} - \boldsymbol{g} - (\boldsymbol{D}+\boldsymbol{D_m})\dot{\boldsymbol{q}} - \boldsymbol{J_t}^T \boldsymbol{f_t} \\ -\dot{\boldsymbol{q}}^T \left[\frac{\partial}{\partial \boldsymbol{q}} \left(\frac{\partial \boldsymbol{C}}{\partial \boldsymbol{q}^T} \right) \right] \dot{\boldsymbol{q}} \\ \boldsymbol{v} - \boldsymbol{R i} - \boldsymbol{K_m} \dot{\boldsymbol{q}} \end{bmatrix} \tag{34}$$

Then Eq (32) can be expressed as

$$\boldsymbol{M}^* \begin{bmatrix} \ddot{\boldsymbol{q}} \\ \boldsymbol{f_n} \\ \dot{\boldsymbol{i}} \end{bmatrix} = \boldsymbol{b} \tag{35}$$

If $\boldsymbol{M}^*$ is confirmed to be nonsingular matrix, and calculate the inverse of $\boldsymbol{M}^*$, then the unknown value of $\ddot{\boldsymbol{q}}$,$\boldsymbol{f_n}$,$\dot{\boldsymbol{i}}$ can be determined based on the above simultaneous equation.

E. The basement model of hyper-redundant mobile manipulator

Translational velocity ${}^W\dot{\boldsymbol{P}}_0$ and angular velocity ${}^W\boldsymbol{\omega}_0$ of link 0 representing mobile robot in $\boldsymbol{\Sigma}_W$ are

$${}^W\dot{\boldsymbol{P}}_0 = {}^W\boldsymbol{R}_0\,{}^0\boldsymbol{V}_0 = {}^W\boldsymbol{R}_0[V_0, 0, 0]^{\mathrm{T}} \tag{36}$$

$${}^W\boldsymbol{\omega}_0 = [0, 0, {}^W\omega_0]^{\mathrm{T}} \tag{37}$$

To consider gravity acceleration into dynamical model, let the gravity vector be defined as ${}^W\boldsymbol{g} = [0,\ 0,\ -g]^{\mathrm{T}}$. Then the translational acceleration ${}^W\ddot{\boldsymbol{P}}_0$ including ${}^W\boldsymbol{g}$ and the angular acceleration ${}^W\dot{\boldsymbol{\omega}}_0$ of link 0 are obtained from Eq.(36) and Eq.(37) as

$${}^W\ddot{\boldsymbol{P}}_0 = {}^W\ddot{\boldsymbol{P}}_0 - {}^W\boldsymbol{g} = {}^W\dot{\boldsymbol{R}}_0\,{}^0\boldsymbol{V}_0 + {}^W\boldsymbol{R}_0\,{}^0\dot{\boldsymbol{V}}_0 - {}^W\boldsymbol{g} \tag{38}$$

$${}^W\dot{\boldsymbol{\omega}}_0 = [0, 0, {}^W\dot{\omega}_0]^{\mathrm{T}} \tag{39}$$

Let ${}^0\boldsymbol{S}_0$ be the gravity center of link 0 with respect to $\boldsymbol{\Sigma}_0$ as shown in Fig.2. By using ${}^0\boldsymbol{S}_0$ and ${}^W\boldsymbol{R}_0$, the position vector ${}^W\boldsymbol{S}_0$ from the origin of $\boldsymbol{\Sigma}_0$ to the gravity center of link 0 with respect to $\boldsymbol{\Sigma}_W$ is expressed as ${}^W\boldsymbol{S}_0 = {}^W\boldsymbol{R}_0\,{}^0\boldsymbol{S}_0$. The acceleration ${}^W\ddot{\boldsymbol{P}}_{G0}$ at the gravity center of link 0 is given with ${}^W\boldsymbol{S}_0$ by

$${}^W\ddot{\boldsymbol{P}}_{G0} = {}^W\ddot{\boldsymbol{P}}_0 + {}^W\dot{\boldsymbol{\omega}}_0 \times {}^W\boldsymbol{S}_0 + {}^W\boldsymbol{\omega}_0 \times ({}^W\boldsymbol{\omega}_0 \times {}^W\boldsymbol{S}_0) \tag{40}$$

and then ${}^W\ddot{\boldsymbol{P}}_{Gi}$ at the gravity center of link i can be calculated by

$${}^W\ddot{\boldsymbol{P}}_{Gi} = {}^W\ddot{\boldsymbol{P}}_i + {}^W\dot{\boldsymbol{\omega}}_i \times {}^W\boldsymbol{S}_i + {}^W\boldsymbol{\omega}_i \times ({}^W\boldsymbol{\omega}_i \times {}^W\boldsymbol{S}_i) \tag{41}$$

The movement of i-th link of manipulator should be given force and torque, so they can be obtained by

$$\begin{aligned} {}^W\boldsymbol{f}_i &= {}^W\boldsymbol{f}_{i+1} + \mathrm{m}_i\,{}^W\ddot{\boldsymbol{P}}_{W,Gi} && (42)\\ {}^W\boldsymbol{n}_i &= {}^W\boldsymbol{n}_{i+1} + {}^W\boldsymbol{I}_i\,{}^W\dot{\boldsymbol{\omega}}_i + {}^W\boldsymbol{\omega}_i \times ({}^W\boldsymbol{I}_i\,{}^W\boldsymbol{\omega}_i) \\ &+ {}^W\boldsymbol{S}_i \times \mathrm{m}_i\,{}^W\ddot{\boldsymbol{P}}_{W,Gi} + {}^W\boldsymbol{P}_{i,i+1} \times {}^W\boldsymbol{f}_{i+1} && (43)\\ \boldsymbol{\tau}_i &= ({}^W\boldsymbol{n}_i^T)\,{}^W\boldsymbol{z}_i + \boldsymbol{I}_{ai}\,\ddot{\boldsymbol{q}}_i + \boldsymbol{C}_i\dot{\boldsymbol{q}}_i && (44) \end{aligned}$$

So the equation of motion of mobile robot that is not carrying any objects can be derived by calculating exerting force and torque on the origin of $\boldsymbol{\Sigma}_0$ from the mobile robot as,

$$\begin{aligned} {}^W\boldsymbol{f}_0 &= \mathrm{m}_0\,{}^W\ddot{\boldsymbol{P}}_{G0} && (45)\\ {}^W\boldsymbol{n}_0 &= {}^W\boldsymbol{I}_0\,{}^W\dot{\boldsymbol{\omega}}_0 + {}^W\boldsymbol{\omega}_0 \times ({}^W\boldsymbol{I}_0\,{}^W\boldsymbol{\omega}_0) \\ &+ {}^W\boldsymbol{S}_0 \times \mathrm{m}_0\,{}^W\ddot{\boldsymbol{P}}_{G0} && (46) \end{aligned}$$

Link 0 can actuate two variables to move from the previous assumptions, which are translation along the direction ${}^W\boldsymbol{x}_0$ and rotation around axis ${}^W\boldsymbol{z}_0$. However, by using non-holonomic constraints, it has three degrees of freedom in the traveling motion. Then the driving force and rotational torque of link 0 are determined by summation and subtraction of left and right wheels' driving torques $\hat{\tau}_L$ and $\hat{\tau}_R$, which are required to make desired rotations of each driving wheel. Taking these relations into consideration, we have two equations between those torques and ${}^W\boldsymbol{f}_0$, ${}^W\boldsymbol{n}_0$ as follows:

$$\begin{aligned} \frac{\hat{\tau}_R}{\mathrm{r}} + \frac{\hat{\tau}_L}{\mathrm{r}} &= {}^W\boldsymbol{f}_0^{\mathrm{T}}\,{}^W\boldsymbol{x}_0 = ({}^W\boldsymbol{R}_0\,{}^0\boldsymbol{f}_0)^{\mathrm{T}}\,{}^W\boldsymbol{R}_0\,{}^0\boldsymbol{x}_0 \\ &= ({}^0\boldsymbol{f}_0^{\mathrm{T}})\,{}^0\boldsymbol{x}_0 = f_0 && (47)\\ \frac{\mathrm{T}}{2}\left(\frac{\hat{\tau}_R}{\mathrm{r}} - \frac{\hat{\tau}_L}{\mathrm{r}}\right) &= {}^W\boldsymbol{n}_0^{\mathrm{T}}\,{}^W\boldsymbol{z}_0 = ({}^W\boldsymbol{R}_0\,{}^0\boldsymbol{n}_0)^{\mathrm{T}}\,{}^W\boldsymbol{R}_0\,{}^0\boldsymbol{z}_0 \\ &= ({}^0\boldsymbol{n}_0^{\mathrm{T}})\,{}^0\boldsymbol{z}_0 = \tau_0 && (48) \end{aligned}$$

Here r is the radius of each wheel and T is the distance between the center of two wheels. $\hat{\tau}_R$ and $\hat{\tau}_L$ are obtained by solving the simultaneous Eqs.(47) and (48). Let $\mathrm{C}_R, \mathrm{C}_L$ and $\mathrm{I}_{aR}, \mathrm{I}_{aL}$ be respectively, viscous damping and inertia moment of right and left driving systems, then the torque τ_R and τ_L that make the traveling motion, that is, the equation of motion are

$$\begin{aligned} \tau_L &= \hat{\tau}_L + \mathrm{I}_{aL}\ddot{q}_L + \mathrm{C}_L\dot{q}_L && (49)\\ \tau_R &= \hat{\tau}_R + \mathrm{I}_{aR}\ddot{q}_R + \mathrm{C}_R\dot{q}_R && (50) \end{aligned}$$

$\hat{\tau}_R$ and $\hat{\tau}_L$, which are calculated from ${}^W\boldsymbol{f}_0$ and ${}^W\boldsymbol{n}_0$, contain $V_0, \dot{V}_0, \omega_0$ and $\dot{\omega}_0$ as variables in them.

III. FORWARD DYNAMICS CALCULATION

To calculate $\boldsymbol{M}^*, \boldsymbol{b}$ in Eq (35), we need to first calculate $\boldsymbol{M}, \boldsymbol{h}, \boldsymbol{g}$. Here we can notice that $\boldsymbol{M}, \boldsymbol{h}, \boldsymbol{g}$ are included Eq (28) that describes the dynamics of non-constraint,and those can be calculated numerically and recursively through forward dynamics calculation [16] by exploiting the inverse dynamics calculation called ''Newton Euler'' Method [17]. Because $\boldsymbol{M}$ is 10×10 matrix when the hyper-redundant manipulator including 10 links, resulting in a large amount of computation to calculate each element of $\boldsymbol{M}$ by using lagrange method. This implies that analytical deriving Eq

(28) is almost impossible by hand writing calculation, then we introduce Newton Euler method as follows.

First of all, Eq (28) should be set as hereunder.

$$\boldsymbol{M_J}\ddot{\boldsymbol{q}} + \boldsymbol{b_J} = \tilde{\boldsymbol{\tau}} \tag{51}$$

Here

$$\begin{aligned} \boldsymbol{M_J} &= \boldsymbol{M}(\boldsymbol{q}) + \boldsymbol{J_m} \\ \boldsymbol{b_J} &= \boldsymbol{h}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q}) + (\boldsymbol{D} + \boldsymbol{D_m})\dot{\boldsymbol{q}} \\ \tilde{\boldsymbol{\tau}} &= \boldsymbol{K_m}\boldsymbol{i} + \boldsymbol{J_c}^T\boldsymbol{f_n} - \boldsymbol{J_t}^T\boldsymbol{f_t} \end{aligned}$$

With forward motion analysis, Eq (51) should be calculated by Newton-Euler method from the bottom link to upper link until the manipulator 's hand, and also with the motion analysis of backward calculation, we get equation of motion of i-th link Eq (52).

$$\tilde{\tau}_i = {}^i\boldsymbol{z}_i^{T\,i}\boldsymbol{n}_i + J_{mi}\ddot{q}_i + (D_i + D_{mi})\dot{q}_i \tag{52}$$

Therefore, the motion Eq (51) can be used to inverse dynamics calculation $\tilde{\boldsymbol{\tau}} = [\tilde{\tau}_1, \tilde{\tau}_2, \cdots, \tilde{\tau}_n]^T$ in Eq (52). This inverse calculation can be described as $\tilde{\boldsymbol{\tau}} = \boldsymbol{p}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}}, \boldsymbol{g})$. Then considering Eq (51) and Eq (52),

$$\boldsymbol{M_J}\ddot{\boldsymbol{q}} + \boldsymbol{b_J} = \boldsymbol{p}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}}, \boldsymbol{g}) \tag{53}$$

Substitute $\ddot{\boldsymbol{q}} = \boldsymbol{0}$ into Eq (53):

$$\boldsymbol{b_J} = \boldsymbol{p}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{0}, \boldsymbol{g}) \tag{54}$$

so $\boldsymbol{b_J}$ can be calculated. Next substitute $\boldsymbol{g} = \boldsymbol{0}, \dot{\boldsymbol{q}} = \boldsymbol{0}, \ddot{\boldsymbol{q}} = \boldsymbol{e}_i (i = 1, 2, \cdots, s)$ into Eq (53), then the $\boldsymbol{b_J} = 0$:

$$\boldsymbol{m}_i = \boldsymbol{M_J}\boldsymbol{e}_i = \boldsymbol{p}(\boldsymbol{q}, \boldsymbol{0}, \boldsymbol{e}_i, \boldsymbol{0}) \tag{55}$$

here we can calculate $\boldsymbol{m}_i$ defined as the component vector of the i-th column in inertia matrix $\boldsymbol{M}$, $\boldsymbol{e}_i$ is a $l \times 1$ matrix in which the i-th element is 1 and others are all 0 like $\boldsymbol{e}_i = [0, 0, \cdots, 1_{(i)}, \cdots, 0, 0]^T$. So with Eq (55) $\boldsymbol{M_J} = [\boldsymbol{m}_1, \boldsymbol{m}_2, \cdots, \boldsymbol{m}_l]$ can be calculated one by one separately.

Thus up to now, we have calculated the $\boldsymbol{M_J}$ and $\boldsymbol{b_J}$. Back to the Eq (33), the $\boldsymbol{M}^*$ can be calculated while the constraint condition is given. Moreover, the inverse of $\boldsymbol{M}^*$ can be also calculated due to invertible for $\boldsymbol{M}^*$.

IV. TRAJECTORY TRACKING SIMULATION

In this section we will introduce the trajectory tracking simulation results. The input voltage of PD controller has been set as follows.

$$\boldsymbol{v} = \boldsymbol{K_p}(\boldsymbol{q_d} - \boldsymbol{q}) + \boldsymbol{K_d}(\dot{\boldsymbol{q}}_{\boldsymbol{d}} - \dot{\boldsymbol{q}}) \tag{56}$$

$\boldsymbol{K_p}$ and $\boldsymbol{K_d}$ are moth $s \times s$ diagonal matrix which indicates a position gain and a velocity gain, $\boldsymbol{q_d}$,$\dot{\boldsymbol{q}}_{\boldsymbol{d}}$ are the desired joint angle and joint angular velocity, respectively.

Simulation's condition has been set as: each link's mass is $m_i = 0.1[kg]$, length is $l_1 = 0[m], l_j = 0.3[m]$,radius of cylindrical link is $r_i = 0.01[m]$,proportional gain is $k_{pi} = 500$,velocity gain is $k_{di} = 20$,viscous friction coefficient of joint is $D_i = 0.5$, torque constant is $K_i = 0.203$, resistance is $R_i = 1.1[\Omega]$, inductance is $L_i = 0.0017[H]$,inertia moment of motor is $I_{mi} = 0.000164$,reduction ratio is $k_i = 3.0$,viscous friction coefficient of reducer is $d_{mi} = 0.01$ and these parameters are given by actucal motor's specifications. Initial condition of each link: $q_1(0) = 0, q_2(0) = 0.25\pi, q_3(0) = 0.5\pi, q_4(0) = -0.5\pi, q_5(0) = 0.25\pi, q_6(0) = 0.25\pi, q_7(0) = -0.5\pi, q_8(0) = 0.25\pi, q_9(0) = -0.25\pi, q_10(0) = 0.25\pi[rad]$,$\dot{q}_i = 0[rad/s]$. Moreover, the desired trajectory's parameters have been set as: $q_{d2}(t) = 0.25\pi, q_{d3}(t) = 0.5\pi, q_{d4}(0) = -0.5\pi, q_{d5}(0) = 0.25\pi, q_{d6}(0) = 0.25\pi, q_{d7}(0) = -0.5\pi, q_{d8}(0) = 0.25\pi[rad]$, and the trajectory has been set as a circle with radius is $0.1[m]$, center is $(x, y, z) = (1.5, 1.5, 0.4)$, target which is tracked by the manipulator hand will rotate in counterclockwise along this circle trajectory. The constraint condition is set as $\boldsymbol{C} = z = 0$. Simulation has been done under three situations and the simulation model is also shown as Fig.3.

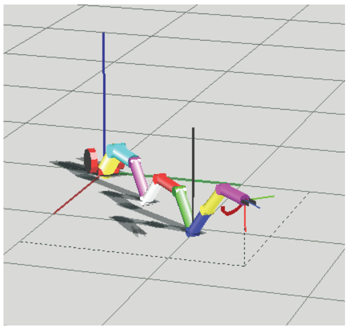


Fig. 3. The simulation model of mobile manipulator

1) trajectory tracking motion with two elbows contacting at joint 4 and joint 7,
2) trajectory tracking motion with just one elbow contacting at joint 7;
3) trajectory tracking motion with just one elbow contacting at joint 4;
4) trajectory tracking motion with no elbow contacting at all.

Simulation results are shown in Fig.4∼Fig.8. Fig.4 shows that manipulator hand's trajectory in xy coordinate under three different simulation condition, in the same way, Fig.6 shows that manipulator hand's trajectory in yz coordinate, and Fig.5 shows that manipulator hand's trajectory in xz coordinate, the simulation time is from 0 to 10[s]. Fig.7 shows that all manipulator links' total amount of work, Fig.8 shows that all manipulator links' total amount of cost electric energy during the whole simulation. From Fig.5, we can tell that the manipulator hand can track the circle trajectory more accurately with more restraint elbows. Especially, in z axis, along with gravity's direction, motion of manipulator hand without restraint elbow is affected by the nutation of each link, the trajectory tracking has not been perfectly accomplished. Moreover, from Fig.7 and Fig.8, even in case of doing same work by manipulator, the cost electric energy is bigger in the motion done by the manipulator without restraint elbow.

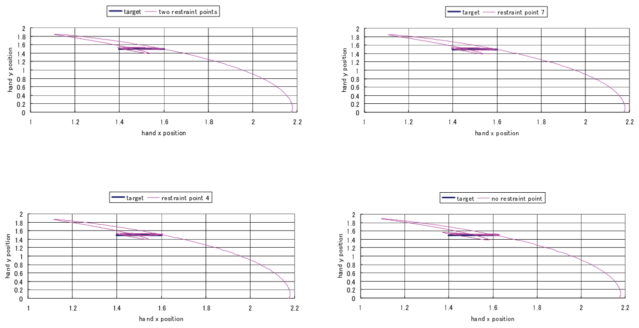


Fig. 4. Trajectory tracking of circle on x-y plane

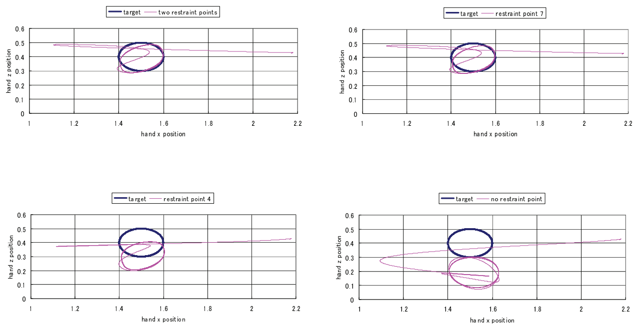


Fig. 5. Trajectory tracking of circle on x-z plane

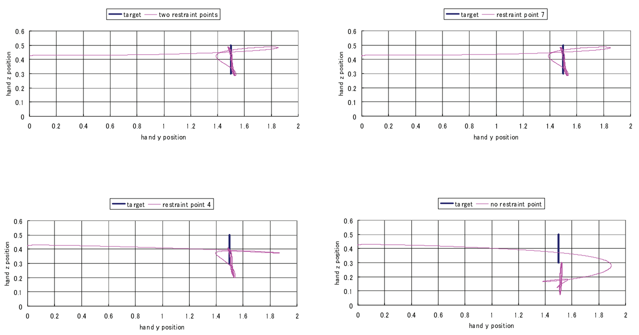


Fig. 6. Trajectory tracking of circle on y-z plane

V. CONCLUSION

We first propose a dynamical model of hyper-redundant mobile manipulator whose plural intermediate links is contacting with environment, second how the contacting raises benefits for saving energy and achieving trajectry tracking accuracy of the end effector. The simulation result shows that less-energy consumption and high trajectry tracking accuracy are achieved by simple PD controller. And the efficiency has been improved drasticaly, and the accuracy also comfirmed to be refined, compared with non-contacting manipulator condition. We think the usage of contacting motion with environment is promising as a new robot motion control strategy.

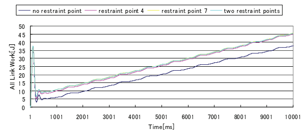


Fig. 7. All Link work

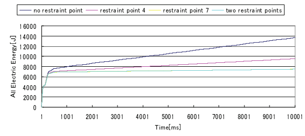


Fig. 8. All Electric energy

REFERENCES

[1] G.S.Chirikjian, J.W.Burdick:''A Hyper-Redundant Manipulator", IEEE Robotics & Automation Magazine, December 1994, pp.22-29 (1994)

[2] F.Fahimi, H.Ashrafiuon and C.Nataraj:''Obstacle Avoidance for Spatial Hyper-Redundant Manipulators", The First Asian Conference on Multibody Dynamics 2002,July 31-August 2, 2002, Iwaki, Fukushima, Japan.

[3] Shugen Ma, Watanabe.M, Kondo.H:''Dynamic control of curve-constrained hyper-redundant manipulators", Computational Intelligence in Robotics and Automation, 2001. Proceedings 2001 IEEE International Symposium, 29 July-1 Aug. 2001 pp,83-88.

[4] Oda.N, Murakami.T, Ohnishi.K,:''A force based motion control strategy for hyper-redundant manipulator", Industrial Electronics, Control and Instrumentation, IECON 97. 23rd International Conference, Volume 3, 9-14 Nov. 1997 pp,1385-1390 vol.3.

[5] Hirose.S, Chu.R:''Development of a light weight torque limiting M-Drive actuator for hyper-redundant manipulator Float Arm ", Robotics and Automation, 1999. Proceedings. 1999 IEEE International Conference , Volume 4, 10-15 May 1999 pp,2831-2836 vol.4 .

[6] Glass K., Colbaugh R., Lim D. and Seraji H, "Real-time collision avoidance for redundant manipulators", *IEEE Transactions on Robotics and Automation*, Vol.11, pp.448-457, 1995.

[7] Homayoun Seraji and Bruce Bon, "Real-Time Collision Avoidance for Position-Controlled Manipulators", *IEEE Transactions on Robotics and Automation*, Vol.15, No.4, pp.670-677, 1999.

[8] Leon Zlajpah and Bojan Nemec, "Kinematic Control Algorithms for On-line Obstacle Avoidance for Redundant Manipulator", *International Conference on Intelligent Robots and Systems*, Lausanne, Vol.2, pp.1898-1903, 2002.

[9] Kwang-Kyu Lee and Martin Buss, "Obstacle Avoidance for Redundant Robots Using Jacobian Transpose Method", *International Conference on Intelligent Robots and Systems*, San Diego, pp.3509-3514, 2007.

[10] Ryo Kurazume and Tsutomu Hasegawa, "Impedance Matching for Free Flying Robots", (in Japanese) *Proc. 20th Annu. Conf. Robot. Soc. Jpn.*, p. 3J16, 2002.

[11] Ryo Kurazume and Tsutomu Hasegawa, "A New Index of Serial-Link Manipulator Performance Combining Dynamic Manipulability and Manipulating Force Ellipsoid", *IEEE Transactions on Robotics*, Vol.22, No.5, pp.1022-1028, 2006.

[12] West,H.,and Asada,H.,''A Method for the Design of Hybrid Position/Force Controllers for Manipulators Constrained by Contact with the Environment", Proc. of 1985 IEEE International Conference on Robotics and Automation, pp.251-260

[13] Jaeheung Park and Oussama Khatib ,''Multi-Link Multi-Contact Force Control for Manipulators", Proc. of 2005 IEEE International Conference on Robotics and Automation, April 2005, pp.3624-3629

[14] Anna Petrovskaya, Jaeheung Park and Oussama Khatib ''Probabilistic Estimation of Whole Body Contacts for Multi-Contact Robot Control", Proc. of 2007 IEEE International Conference on Robotics and Automation, April 2007, pp.568-573

[15] Takeshi Ikeda and Mamoru Minami:''Research of Grinding Robot without Force Sensor by Using Algebraic Equation"(in Japanese), Transactions of the Japan Society of Mechanical Engineers(C), Vol.71, No.702, pp.270-277, 2005.

[16] M.W.Walker and D.E.orin:''Efficient Dynamic Computer Simulation of Robotic Mechanisms", ASME J.of DSMC,104, pp.205-211, 1982

[17] L.R.Hunt, R.Su and G.Meyer:''Global Transformation of Non Linear system," IEEE Trans. AC., 28-1, pp.24-31, 1983