Comparison of Frequency Responses with PID and Two Degree-of-Freedom PID Control for Underactuated Flying Object

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Abstract: Flying object used in our research is an experimental device like a helicopter. Helicopter, which has a special characteristics such as hovering, vertical ascent and vertical dive. Therefore, it is widely used at the time of disaster. For example, manned helicopter is used for rescue work, emergency transport and fire fighting. On the contrary, unmanned helicopter is used to get precious information in a danger area for human. But there are problems that the structure is complicated, it has a high nonlinearity. And it is difficult to keep the attitude, because it is influenced by wind in flight. An experimental device in our research has an underactuated system having 2-input and 3-output that has number of input less than number of output. So, this system is able to reduce cost, but to keep the posture is difficult. Therefore, in our research, a stabilization method called two degree-of-freedom PID control has been proposed. It is the system having three degree-of-freedom. Although the effectiveness has been confirmed through the simulation and experimental results in time-domain, the characteristics of the proposed method was just explored by the result of output for step-type reference signal. Therefore, this paper confirms the effectiveness of the proposed method through the simulation and experimental result in frequency-domain in order to check the characteristics of the proposed method.

Keywords: Underactuated flying object, Two degree-of-freedom PID, Frequency response

1. INTRODUCTION

The helicopter is applied in large field because of flight ability such as vertical ascent, vertical dive and hovering. Especially manned helicopter is used for rescue, emergency activity and fire fighting at the time of disaster, and unmanned helicopter is precious sources of information in the danger area where people cannot approach. But structure of helicopter is complex and sensitive to the influence of the wind. There are mainly two models in a helicopter. One is a single-rotor helicopter having main rotor and tail rotor. Tail rotor is to generate anti torque. Another one is a twin-rotor helicopter having two main rotors. In this works, it especially considers about twin-rotor helicopter. And this model is better than single rotor helicopter about the operational stability of roll direction.

Our laboratory has an experimental device of three degree-of-freedom underactuated flying object like twin-rotor helicopter. Twin-rotor helicopter has an advantage in the safety, because this model has a characteristic that operation stability against roll is high. This device controls the angles of vertical and rotation direction by thrust gained by two rotors. Controlling an underactuated flying object has attracted a lot of attention, due to the fact that flying object is an underactuated nonlinear system. It is considered that it is possible to contribute for reducing weight, lowering the cost, and the energy saving if the system can be controlled with the number of control inputs less than the number of the system outputs.

In previous researches [1]-[3], nonlinear controllers were proposed. On the other hand, in this research, PID control is used in order to design the controller of nonlinear model. Although, PID control is generally used, we proposed a method named as two degree-of-freedom PID control which aims to improve PID control. The previous research [4] verified static characteristics that tracking a certain reference signal by PID control and two degree-of-freedom PID control. So, it verifies an availability of two degree-of-freedom PID control's efficiency. And this research verifies dynamics characteristics of an underactuated flying object by experiments on frequency responses. This paper organizes as follows. In section 2, we represent modeling of an underactuated flying object. In section 3, we design PID control system and two degree-of-freedom PID control system based on section 2. Section 4 shows results of experiments, and finally we conclude this paper.

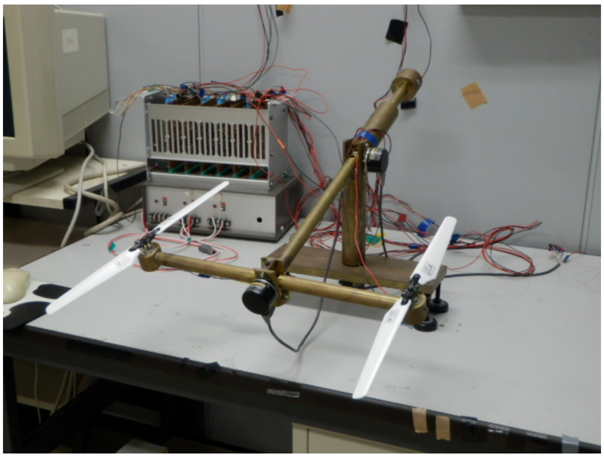

Fig. 1 Underactuated flying object

2. MODELING

Controlled target is three degree-of-freedom underactuated flying object as shown Fig. 1. It is a two inputs and three outputs system which attaches motors for turn-

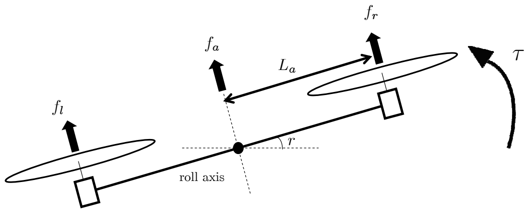

Fig. 2 Roll direction

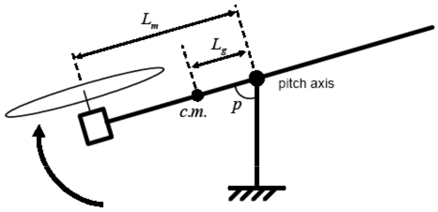

Fig. 3 Pitch direction

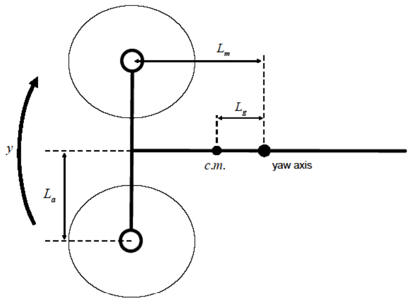

Fig. 4 Yaw direction

ing left and right rotor, and rotary encoders for detecting roll angle, pitch angle and yaw angle. To keep flying object from spinning by rotor drag torque, rotation of the right rotor is reverse rotation of the left rotor. The equation of motion of three degree-of-freedom underactuated flying object is shown as follows and indicates the coupling between each axis.

Direction of the roll angle:

$$I_r\ddot{r} + D_r\dot{r} = \tau \tag{1}$$

Direction of the pitch angle:

$$I_p\ddot{p} + D_p\dot{p} + mgL_g \sin p = L_m f_a \cos r \tag{2}$$

Direction of the yaw angle:

$$I_y\ddot{y} + D_y\dot{y} = L_m f_a \sin r \tag{3}$$

Where r, p and y are angles of each direction, m is weight of the system, g is gravity acceleration, I_r, I_p and I_y are moments of inertia of each direction, D_r, D_p and D_y are coefficient of friction of each direction, L_m is distance from pitch axis to roll link and L_g is distance from pitch axis to center of mass.

f_a is a resultant force of f_l and f_r, τ is a moment of roll direction.

$$f_a = f_r + f_l \tag{4}$$
$$\tau = L_a(f_l - f_r) \tag{5}$$

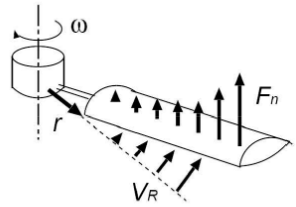

Fig. 5 Forces which act on the face of rotor

Where f_r and f_l are right rotor thrust and left one respectively. L_a is arm length from roll axis to the motor.

$$f_r = \omega_r^2 A = A(ku_r)^2 = Ak^2u_1 \tag{6}$$
$$f_l = \omega_l^2 A = A(ku_l)^2 = Ak^2u_2 \tag{7}$$

Where $\omega_{r,l}$ are rotor angular speeds, A is a coefficient based on shape of the rotor, $u_{r,l}$ are input voltages into each left and right motor, k is a coefficient between voltage and angular speed, where $\omega_r = ku_r$ and $\omega_l = ku_l$.

2.1 Aerodynamical forces

The state of the flying object is shifted to the desired state by aerodynamical forces which act on the rotors. The equation of aerodynamical forces is shown by using the rotor angular speed as follows. Aerodynamical force per microscopic area is shown as follows.

$$F_n = \frac{1}{2}\rho V_R^2 S C_z \tag{8}$$
$$V_R = \omega r \tag{9}$$

Where F_n is aerodynamical force per microarea, ρ is airdensity, V_R is airspeed, S is surface area of the rotor, C_z is a coefficient of aerodynamical forces and r is distance from shaft. F_n is a function of r as shown in Fig. 5. Airdensity ρ and airspeed V_R are variables. Surface area of the rotor S, shape of rotor and rotor area which affect C_z are constants. As a result, force of aerodynamical forces per microscopic area is the rotor thrust F_N,

$$\begin{aligned} F_N &= 2\int_0^R F_n dr \\ &= \int_0^R \rho(r\omega)^2 S C_z dr \\ &= \omega^2 S \int_0^R \rho r^2 C_z dr \\ &= \omega^2 A \end{aligned} \tag{10}$$

Coefficient based on shape of rotor A is

$$A = S\int_0^R \rho r^2 C_z dr \tag{11}$$

Where R is a radius of the rotor.

2.2 Input voltage

Input voltage sets a limit because of hardware's specification.

$$0[\mathrm{V}] \le u_r \le 7[\mathrm{V}] \tag{12}$$
$$0[\mathrm{V}] \le u_l \le 7[\mathrm{V}] \tag{13}$$

3. CONTROL SYSTEM DESIGN

3.1 PID control design

Based on equation of motion which shows the relationship between input voltage and rotor angular speed, equation of motion is given as follows.

$$
\begin{aligned}
I_r\ddot{r} + D_r\dot{r} &= L_a A k^2 (u_2 - u_1) \\
I_p\ddot{p} + D_p\dot{p} + mgL_g \sin p &= L_m A k^2 (u_1 + u_2)\cos r \\
I_y\ddot{y} + D_y\dot{y} &= L_m A k^2 (u_1 + u_2)\sin r
\end{aligned} \quad (14)
$$

Where u_1 is the square of u_r, and u_2 is the square of u_l. Parameters of the equations are replaced and shown as follows.

$$
\begin{aligned}
a_1\ddot{r} + a_2\dot{r} &= u_2 - u_1 \\
b_1\ddot{p} + b_2\dot{p} + b_3 \sin p &= (u_1 + u_2)\cos r \\
c_1\ddot{y} + c_2\dot{y} &= (u_1 + u_2)\sin r
\end{aligned} \quad (15)
$$

Where

$$
\begin{aligned}
a_1 &= \frac{I_r}{L_a A k^2} & b_1 &= \frac{I_p}{L_m A k^2} \\
a_2 &= \frac{D_r}{L_a A k^2} & b_2 &= \frac{D_p}{L_m A k^2} \\
c_1 &= \frac{I_y}{L_m A k^2} & b_3 &= \frac{mgL_g}{L_m A k^2} \\
c_2 &= \frac{D_y}{L_m A k^2}
\end{aligned}
$$

From (15), defining that;

$$
\begin{aligned}
F_r &= \frac{1}{a_1}\left\{-a_2\dot{r} + (u_2 - u_1)\right\} \\
F_p &= \frac{1}{b_1}\left\{-b_2\dot{p} - b_3\sin p + (u_1 + u_2)\cos r\right\} \\
F_y &= \frac{1}{c_1}\left\{-c_2\dot{y} + (u_1 + u_2)\sin r\right\}
\end{aligned} \quad (16)
$$

Moreover, we design the desired F_p^*, F_y^* as follows.

$$F_p^* = \frac{1}{b_1}\left(-b_2\dot{p} - b_3\sin p + z_p\cos r\right) \quad (17)$$

$$F_y^* = \frac{1}{c_1}\left(-c_2\dot{y} + z_y\sin r\right) \quad (18)$$

Where z_p and z_y are ideal input voltages replaced u_1 and u_2. From (17) and (18), we can obtain the desired roll angle as follows.

$$r^* = \tan^{-1}\left(\frac{c_1}{b_1}\frac{F_y^* + \frac{c_2}{c_1}\dot{y}}{F_p^* + \frac{b_2}{b_1}\dot{p} + \frac{b_3}{b_1}\sin p}\right) \quad (19)$$

And the desired input voltages F_r^*, F_p^* and F_y^* are given by the following PID control.

$$
\begin{aligned}
F_r^* &= -K_{P1}(r - r^*) - K_{I1}\int (r - r_d) \\
&\qquad -K_{D1}(\dot{r} - \dot{r_d}) \\
F_p^* &= -K_{P2}(p - p_d) - K_{I2}\int (p - p_d) \\
&\qquad -K_{D2}(\dot{p} - \dot{p_d}) \\
F_y^* &= -K_{P3}(y - y_d) - K_{I3}\int (y - y_d) \\
&\qquad -K_{D3}(\dot{y} - \dot{y_d})
\end{aligned} \quad (20)
$$

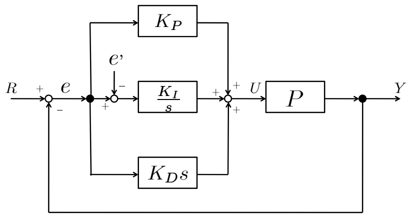

Fig. 6 Two degree-of-freedom PID control structure

Where defining as;

$$u_2 - u_1 = a_1 F_r^* + a_2\dot{r} = z_1 \quad (21)$$

$$u_1 + u_2 = \frac{b_1 F_p^* + b_2\dot{p} + \sin p}{\cos r} = z_2 \quad (22)$$

From (21) and (22), u_1 and u_2 are obtained as follows.

$$u_1 = \frac{z_2 - z_1}{2} \quad (23)$$

$$u_2 = \frac{z_1 + z_2}{2} \quad (24)$$

Because $u_1 = u_r^2$ and $u_2 = u_l^2$, u_r and u_l are given as follows.

$$u_r = \sqrt{u_1} \quad (25)$$

$$u_l = \sqrt{u_2} \quad (26)$$

Since this device cannot carried out the reverse rotation, u_r and u_l are only positive signal. If u_r and u_l are negative signal, u_r and u_l are set to be zero.

3.2 Two degree-of-freedom PID control design

Two degree-of-freedom PID control law aims to adjust target-tracking and disturbance-reduction ability independently.

Two degree-of-freedom PID control structure is shown in Fig. 6. Where Y is output such as a present angle, U is control input obtained by each gain, R is reference signal of each angle, $e^{'}$ is target-tracking error with PD control using the same gain as PID control. The difference between e and $e^{'}$ is applied to the integrator, that is, if there is neither disturbance nor modeling error, the integral compensation doesn't appear. Then the proposed controller is given as follows.

$$
\begin{aligned}
F_p^* &= -K_{P2}(p - p_d) - K_{I2}\int \left((p - p_d) - e_p^{'}\right) \\
&\qquad -K_{D2}(\dot{p} - \dot{p_d}) \\
F_y^* &= -K_{P3}(y - y_d) - K_{I3}\int \left((y - y_d) - e_y^{'}\right) \\
&\qquad -K_{D3}(\dot{y} - \dot{y_d})
\end{aligned}
$$

In this controller, the effect of integral compensation performs only when there is disturbance or modeling error.

4. EXPERIMENT

The previous research [4] have compared the tracking performances for step-type reference signal with PID and two degree-of-freedom PID in time domain, in order to verify the validity of two degree-of-freedom PID

Table 1 Model parameters

a_1 = 15.88	b_1 = 43.69
a_2 = 1.02	b_2 = 1.02
c_1 = 24.69	b_3 = 36.11
c_2 = 1.84	

Table 2 Control parameters

K_{P1} = 2.9	K_{I3} = 0.001
K_{P2} = 4.1	K_{D1} = 2.5
K_{P3} = 4.7	K_{D2} = 5.0
K_{I1} = 0.00001	K_{D3} = 5.5
K_{I2} = 0.01	

Table 3 Control parameters

K_{P1} = 5.0	K_{I3} = 0.001
K_{P2} = 9.4	K_{D1} = 2.5
K_{P3} = 38.0	K_{D2} = 5.0
K_{I1} = 0.001	K_{D3} = 5.5
K_{I2} = 0.01	

Table 4 Control parameters

PID control(pitch)	-3.12
Two degree-of-freedom PID control(pitch)	-3.33
PID control(yaw)	-5.88
Two degree-of-freedom PID control(yaw)	-8.93

for underactuated flying object. On the other hand, this paper aims to clarify the characteristics of two degree-of-freedom PID control system through the frequency response experiment. Therefore this section shows an experimental results of frequency response for pitch and yaw direction in case of using two degree-of-freedom PID control. The model parameters of experimental system (15) shown in Fig.1 are given in Table 1.

4.1 Frequency responses for pitch direction

We set up a sine wave shown in (27) as reference signal, in order to carry out frequency response experiments for pitch direction. And we measured the output response using five patterns of frequency ω shown in a equation (28). Moreover we define the steady state value of output response after 20 seconds as the amplitude value, in order to draw gain diagram.

$$P_d = \frac{\pi}{2} + 0.1 \sin \omega t \quad (27)$$

$$\omega = \left[\begin{array}{ccccc} 0.03 & 0.1 & 0.3 & 1.0 & 3.0 \end{array} \right] \quad (28)$$

In frequency response experiments for pitch direction, we use control parameters shown in Table 2 to compare controlled performances of PID control method and proposed method. Experimental results are shown in Fig. 7 and Fig. 8. It turns out that each gain diagram is similar. And summation of gains derived from each experiment is shown in Table 4. Although the summation of gains for pitch direction experiments in the proposed method is smaller than PID control system's result, the derived gains at ω = 0.03, 0.1, 0.3 and 1.0 in the proposed method are larger than PID control system's result in Fig.7 and Fig.8. It means that the proposed method has the better tracking performance than PID control method for ω = 1.0 or less. Therefore we believe that the proposed method outperforms PID control method in terms of the tracking performance at low frequency band.

4.2 Frequency responses for yaw direction

Next, frequency response experiments for yaw direction is described. We set up a sine wave shown in (29) as the reference signal, and frequency ω is set up as five patterns in (30). And we used the steady state value of output response after 25 seconds as the amplitude value in order to show gain diagram. And the parameter of the experiment of yaw axis has to change because that of pitch axis is not able to track to the target value.

$$Y_d = -0.3 \sin \omega t \quad (29)$$

$$\omega = \left[\begin{array}{ccccc} 0.01 & 0.02 & 0.03 & 0.07 & 0.1 \end{array} \right] \quad (30)$$

In frequency response experiments for yaw direction, we use control parameters shown in Table 3 to confirm the effectiveness of proposed method. The Experimental results are shown in Fig.9 and Fig.10. In low frequency (e.g. ω = 0.01) band, the proposed method outperforms PID control in terms of the tracking performance because the gain derived by the proposed method (e.g. -0.30[dB]) is larger than PID's result (e.g. -0.32[dB]). However, in other frequency band, tracking performance of PID control was better than proposed method.

5. CONCLUSIONS

This paper compared the experimental results of frequency response with PID and two degree-of-freedom PID control method for underactuated flying object. In the previous research [4], although the experiments on tracking to step-type reference signal have been explored in order to verify the validity of the proposed method in time domain, tracking performance to arbitrary reference signal was not considered. Therefore this paper explored the gain diagrams of frequency response and the tracking performance using the proposed method for an underactuated flying object. When the same control parameters for PID and two degree-of-freedom PID controller are used, the effectiveness of the proposed method was confirmed about pitch direction. Moreover, for yaw direction, the effectiveness of the proposed method was confirmed in low frequency band. However there are the cases that PID control method outperforms the proposed method. In this work, the result that two degree-of-freedom PID control is better than PID control isn't able to show. As future works, there are an experiment

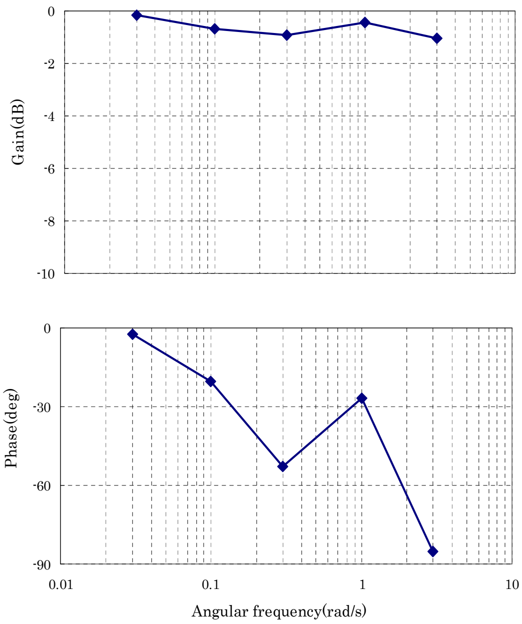

Fig. 7 Bode diagram of PID control in pitch direction

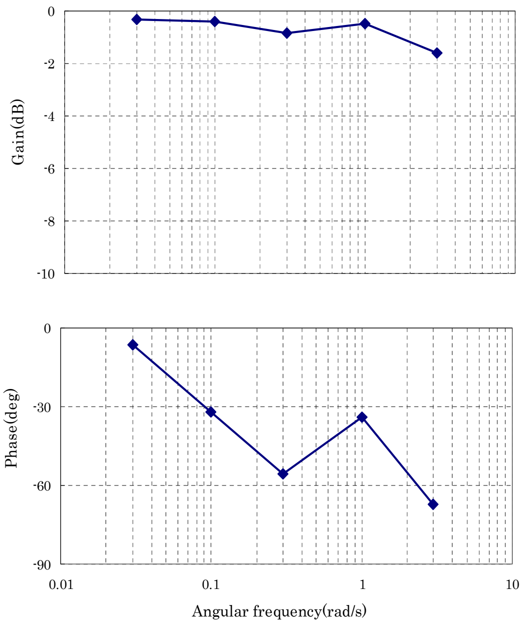

Fig. 8 Bode diagram of two degree-of-freedom PID control in pitch direction

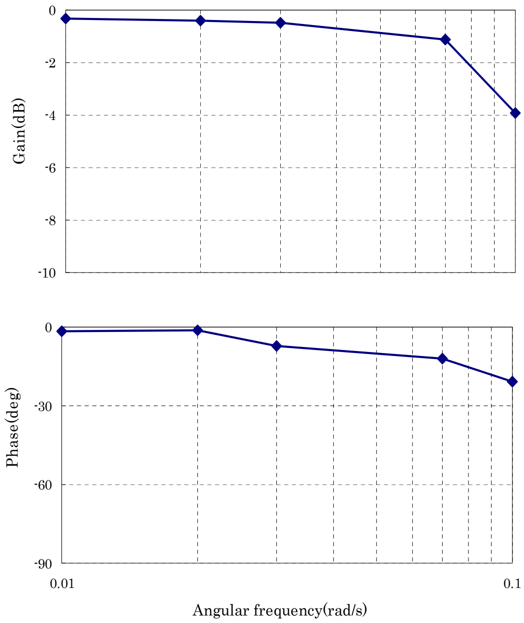

Fig. 9 Bode diagram of PID control in yaw direction

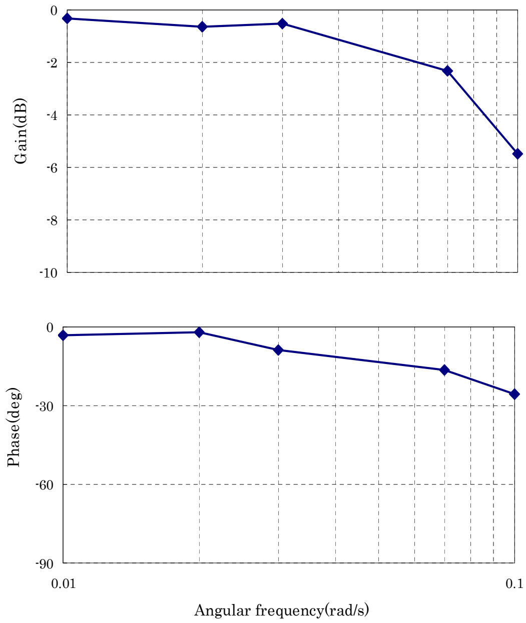

Fig. 10 Bode diagram of two degree-of-freedom PID control in yaw direction

and comparison in case of using design parameters which show the best performances.

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