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# 冗長マニピュレータの動的形状変更可操作性

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## Dynamic Reconfiguration Manipulability Analysis of Redundant Robot

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In this paper, we propose a new index inspired by dynamic manipulability for redundent robot proposed by Yoshikawa to estimate dynamic ability to change configuration by using remaining redundancy, while prior task is controlled. Several indexes have been proposed so far to measure statical and dynamical capability of robot manipulator. So, we proposed a new concept named Dynamic Reconfiguration Manipulability (DRM) which combined dynamic manipulability with reconfiguration manipulability.

***Key Words*** : Dynamic, reconfiguration, acceleration

## 1. Introduction

A variety of researches that discussed about performance of robot manipulators were extended from the middle of 1980 's to the end of 1990 's[(1)(2)(3)]. To evaluate these manipulabilities, ellipsoids are used showing velocity's directions and values. On the other hand, Reonfiguration Manipulability of redundant manipulators was proposed using kinematics to indicate intermediate links' manipulabilities[(4)], and its usefulness to obstacle avoidance and singular configuration avoidance had been improved. However, as is well-known, with the increase of degrees of redundant manipulators, weights are also increasing. So, in this situation, it is not suitable for only basing on kinematics when high-accuracy control is required.

Therefore, we propose a new concept Dynamic Reconfiguration Manipulability(DRM) which is a measure of how much a dynamical system can potentially produce a motion in a workspace with normalized input torque, by combining the Dynamic Manipulability with Reconfiguration Manipulability. In this paper, we propose the concept of DRM, and prove physical properties, validity and usefulness of DRM using 4 links manipulator in 2-dimensional space.

## 2. Dynamic Reconfiguration Manipulability

### 2·1 Dynamic Manipulability

In general, equation of dynamics for serial link manipulators is written as

$$M(q)\ddot{q} + h(q,\dot{q}) + g(q) + D\dot{q} = \tau \tag{1}$$

where $M(q) \in \mathrm{R}^{n\times n}$ is inertia matrix, $h(q,\dot{q}) \in \mathrm{R}^n$ and $g(q) \in \mathrm{R}^n$ are vectors which indicate Coriolis force, centrifugal force and gravity, $D = diag[d_1, d_2, \cdots, d_n]$ is matrix which means coefficients of joints' viscous friction and $\tau \in \mathrm{R}^n$ is joint torque. The kinematic equation of a robot, that the relation between the $i$-th link's velocity $\dot{r}_i \in \mathrm{R}^m$ and the angular

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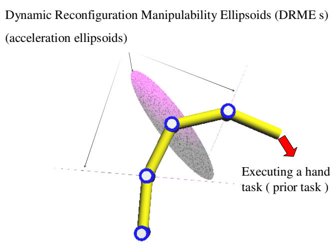


Fig. 1 Application of Dynamic Reconfiguration Manipulability for redundant manipulator

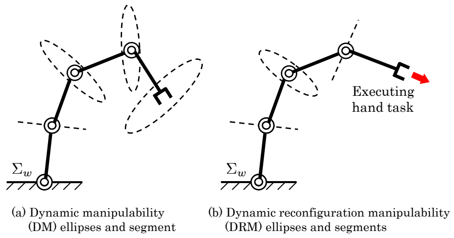


Fig. 2 (a) Dynamic manipulability ellipsoids (DMEs), (b) dynamic reconfiguration manipulability ellipsoids (DRMEs)

velocity $\dot{q} \in \mathrm{R}^n$ is represented by

$$\dot{r}_i = J_i \dot{q} \quad (i = 1, 2, \cdots, n) \tag{2}$$

Here, $J_i \in R^{m \times n}$ can be described as Jacobian matrix with zero block matrix, $J_i = [\tilde{J}_i,\ 0]$. By differentiating Eq. (2), we can obtain the following equation.

$$\ddot{r}_i = J_i(q)\ddot{q} + \dot{J}_i(q)\dot{q} \tag{3}$$

where we can understand that $\dot{J}_i(q)\dot{q}$ is the acceleration as Coliolis and centrifugal acceleration resulted from nonlinear relation of two-coordinates-space represented by $q_i$ to $r_i$. Then, from Eqs. (1) and (3) we can obtain the following equation.

$$\ddot{r}_i - \dot{J}_i(q)\dot{q} = J_i M^{-1}[\tau - h(q, \dot{q}) - g(q) - D\dot{q}] \tag{4}$$

Here, two variables are defined as follows:

$$\tilde{\tau} \stackrel{\triangle}{=} \tau - h(q, \dot{q}) - g(q) - D\dot{q} \tag{5}$$

$$\tilde{\ddot{r}}_i \stackrel{\triangle}{=} \ddot{r}_i - \dot{J}_i(q)\dot{q} = J_i(q)\ddot{q} \tag{6}$$

So, Eq. (4) can be rewritten as

$$\tilde{\ddot{r}}_i = J_i M^{-1}\tilde{\tau} \quad (i = 1, 2, \cdots, n) \tag{7}$$

Considering desired accelerations $\tilde{\ddot{r}}_{id}$ of all links yielded by a set of joint torques $\tilde{\tau}$ which satisfies an Euclidean norm condition, that is, $\|\tilde{\tau}\| = (\tilde{\tau}_1^2 + \tilde{\tau}_2^2 + \cdots + \tilde{\tau}_n^2)^{1/2} \leq 1$, each tip acceleration shapes an ellipsoid in range space of $J_i$. These ellipsoids of each link have been known as “Dynamic Manipulability Ellipsoid (DME)” (2) (Fig.2(a)) which are described as

$$\tilde{\ddot{r}}_{id}^{\,T}[J_i(M^T M)^{-1} J_i^T]^{+}\tilde{\ddot{r}}_{id} \leq 1, \quad and \quad \tilde{\ddot{r}}_{id} \in \mathrm{R}(J_i) \tag{8}$$

where, $\mathrm{R}(J_i)$ represents range space of $J_i$.

### 2·2 Dynamic Reconfiguration Manipulability

Here we assume that the desired end-effector's acceleration $\ddot{r}_{nd}$ is given as primary task. The relation between $\tilde{\ddot{r}}_n$ and $\tilde{\tau}$ is denoted to change $i$ to $n$ from Eq. (7), then,

$$\tilde{\ddot{r}}_n = J_n M^{-1}\tilde{\tau} \tag{9}$$

Solving Eq. (9) for $\tilde{\tau}$ yielding desired acceleration $\tilde{\ddot{r}}_{nd}$

$$\tilde{\tau} = (J_n M^{-1})^{+}\tilde{\ddot{r}}_{nd} + [I_n - (J_n M^{-1})^{+}(J_n M^{-1})]\,{}^1 l \tag{10}$$

${}^1 l$ is an arbitrary vector satisfying ${}^1 l \in \mathrm{R}^n$. The left superscript “1” of ${}^1 l$ means the first dynamic reconfiguration task. In the right side of Eq. (10), the first term denotes the solution making $\tilde{\tau}$ minimize in the null space of $J_n M^{-1}$ during implementing $\ddot{\bar{r}}_{nd}$. The second term denotes the components of torques at each joint, which can change the shape of manipulator regardless with the influence of $\ddot{\bar{r}}_{nd}$ given arbitrarily as end-effector acceleration for tracking the desired trajectory. Providing the first dynamic reconfiguration task, that is the first reconfiguration task ${}^1\ddot{\bar{r}}_{jd}$ $(j = 1,2,\cdots,n-1)$, is given to the $j$-th link, shall we discuss realizability of ${}^1\ddot{r}_{jd}$ in the following argument. In this research, ${}^1\ddot{r}_{jd}$ is assumed to be commanded by an dynamic reconfiguration control system of higher level. We can obtain the relation of ${}^1\ddot{\bar{r}}_j$ and $\ddot{\bar{r}}_{nd}$ from Eqs. (7) and (10).

$$ {}^1\ddot{\bar{r}}_j = J_jM^{-1}(J_nM^{-1})^{+}\ddot{\bar{r}}_{nd} + J_jM^{-1}[I_n - (J_nM^{-1})^{+}(J_nM^{-1})]\,{}^1l \tag{11} $$

Then, according to the relation of Eq. (6),

$$ {}^1\ddot{r}_j - \dot{J}_j\dot{q} - J_jM^{-1}(J_nM^{-1})^{+}(\ddot{r}_{nd} - \dot{J}_n\dot{q}) = J_jM^{-1}[I_n - (J_nM^{-1})^{+}(J_nM^{-1})]\,{}^1l \tag{12} $$

Here, we define three variables shown as

$$ {}^1\ddot{\hat{r}}_j \stackrel{\triangle}{=} \dot{J}_j\dot{q} + J_jM^{-1}(J_nM^{-1})^{+}(\ddot{r}_{nd} - \dot{J}_n\dot{q}) \tag{13} $$

$$ \Delta{}^1\ddot{r}_j \stackrel{\triangle}{=} {}^1\ddot{r}_j - {}^1\ddot{\hat{r}}_j \tag{14} $$

$$ {}^1\Lambda_j \stackrel{\triangle}{=} J_jM^{-1}[I_n - (J_nM^{-1})^{+}(J_nM^{-1})] \tag{15} $$

In Eq. (13), ${}^1\ddot{\hat{r}}_j$ is represented for the acceleration caused by manipulator's shape-changing. In the right side of Eq. (13), the first term is denoted as the Coliolis and centrifugal acceleration of $j$-th link, and the second term is represented for the influence of ${}^1\ddot{\bar{r}}_{nd}$ on $j$-th link except Coliolis and centrifugal acceleration at $n$-th link because its acceleration is peculiar on its coordinate flame. So, Eq. (12) can be rewritten as

$$ \Delta{}^1\ddot{r}_j = {}^1\Lambda_j\,{}^1l \tag{16} $$

The relation between ${}^1\ddot{r}_j$ and $\Delta{}^1\ddot{r}_j$ is shown in Fig. 3. However, the problem is whether we can yield desired $\Delta{}^1\ddot{r}_{jd}$, that is, whether we can find ${}^1l$ to generate $\Delta{}^1\ddot{r}_{jd}$. From Eq. (16), we can obtain ${}^1l$ as

$$ {}^1l = {}^1\Lambda_j^{+}\Delta{}^1\ddot{r}_{jd} + (I_n - {}^1\Lambda_j^{+}\,{}^1\Lambda_j)\,{}^2l \tag{17} $$

In Eq. (17), ${}^2l$ is an arbitrary vector satisfying ${}^2l \in \mathrm{R}^n$. Assuming that ${}^1l$ is restricted as $\|{}^1l\| \le 1$, then we obtain next relation,

$$ (\Delta{}^1\ddot{r}_{jd})^T({}^1\Lambda_j^{+})^T\,{}^1\Lambda_j^{+}\Delta{}^1\ddot{r}_{jd} \le 1 \tag{18} $$

If $rank({}^1\Lambda_j) = m$, Eq. (18) represents an ellipsoid expanding in $m$-dimensional space, holding

$$ \Delta{}^1\ddot{r}_{jd} = {}^1\Lambda_j\,{}^1\Lambda_j^{+}\Delta{}^1\ddot{r}_{jd}, \quad \Delta{}^1\ddot{r}_{jd} \in \mathrm{R}^m, \tag{19} $$

which indicates that $\Delta{}^1\ddot{r}_{jd}$ can be arbitrarily generated in $m$-dimensional space and Eq. (16) always has the solution ${}^1l$ corresponding to all $\Delta{}^1\ddot{r}_{jd} \in \mathrm{R}^m$. On the other hand, if $rank({}^1\Lambda_j) = r < m$, $\Delta\ddot{r}_{jd}$ does not value arbitrarily in $\mathrm{R}^m$. In this case, reduced $\Delta\ddot{r}_{jd}$ is denoted as $\Delta{}^1\ddot{r}^*_{jd}$. Then Eq. (18) is written as

$$ (\Delta{}^1\ddot{r}^*_{jd})^T({}^1\Lambda_j^{+})^T\,{}^1\Lambda_j^{+}\Delta{}^1\ddot{r}^*_{jd} \le 1, (\Delta{}^1\ddot{r}^*_{jd} = {}^1\Lambda_j\,{}^1\Lambda_j^{+}\Delta{}^1\ddot{r}_{jd}) \tag{20} $$

Equation (20) describes an ellipsoid expanded in $r$-dimensional space. These ellipsoids of Eqs. (18) and (20) are shown in Fig.2(b).

### 2·3 Dynamic Reconfiguration Manipulability Shape Index(DRMSI)

In this section, we propose the index evaluating DRM. Thus, by applying the singular value decomposition for this matrix $\Lambda$, we get

$$ {}^1\Lambda_j = {}^1U_j^1\Sigma_j^1V_j^T \tag{21} $$

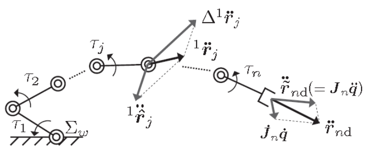



Fig. 3 Reconfiguration at intermediate link during hand task executed

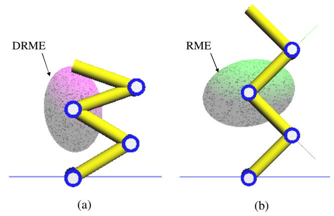



Fig. 4 Shapes of manipulator: (a) shape of the peak of 2nd-link's DRMM with DRME, (b) shape of the peak of 2nd-link's RMM with RME

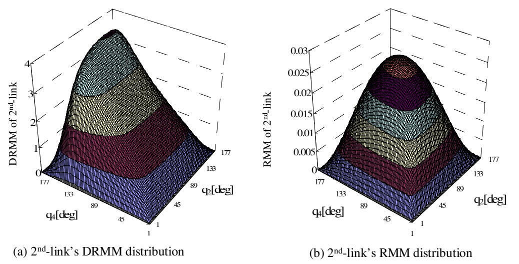



Fig. 5 2nd-link's DRMM and RMM distribution

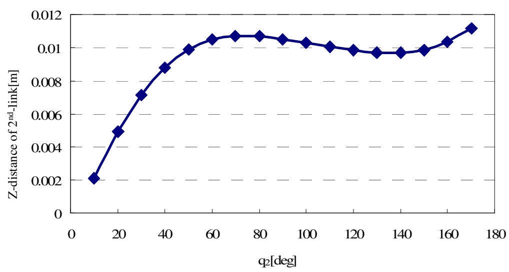



Fig. 6 Z-direction's moving distance of 2nd-link

$$
{}^1\Sigma_j = \begin{array}{r} \\ r \\ \\ m-r \end{array} \begin{array}{c} \begin{array}{cc} \quad r \quad & \quad n-r \quad \end{array} \\ \left[ \begin{array}{ccc|c} {}^1\sigma_{j,1} & & 0 & \\ & \ddots & & 0 \\ 0 & & {}^1\sigma_{j,r} & \\ & 0 & & 0 \end{array} \right] \end{array} \tag{22}
$$

In Eqs. (21) and (22), ${}^1U \in \mathrm{R}^{m\times m}, {}^1V \in \mathrm{R}^{n\times n}$ are orthogonal matrixes, and $r$ denotes the number of non-zero singular values of ${}^1\Lambda_j$ and $\sigma_{j,1} \geq \cdots \geq \sigma_{j,r} > 0$. In addition, $r \leq m$ because $rank({}^1\Lambda_j) \leq m$. So, dynamic reconfiguration capability of $j$-th link when hand of manipulator operating task can be described by following equation.

$$
{}^1w_j = {}^1\sigma_{j,1} \cdot {}^1\sigma_{j,2} \cdots {}^1\sigma_{j,r} \tag{23}
$$

In this paper, we defined the value of $w_j$ in Eq. (23) as dynamic reconfiguration manipulability measure (DRMM), which indicates the degree of that reconfiguration acceleration of $j$-th link can be generated for arbitrary direction. And, volume of dynamic reconfiguration ellipsoid at the $j$-th link is described as ${}^1V_{DRj}$. Then, in order to consider dynamic reconfiguration measure of the whole manipulator-links, we define a index named dynamic reconfiguration manipulability shape index (DRMSI) as follows:

$$
{}^1W_{DR} = \sum_{j=1}^{n-1} a_j \, {}^1V_{DRj} \tag{24}
$$

Here, $a_j$ is unit adjustment between different dimension. In this paper, singular-values increase a hundredfold to enlarge value of ellipsoid, compared to ellipse or line segment.

## 3. Simulations

In this section, we use a 4-links redundant manipulator which works in 2-dimentional space to confirm the physical properties of DRM, and prove the validity and usefulness of DRM. Length, mass and coefficient of viscous friction of each link are set to be 0.3[m], 1.0[kg], and 2.0[N·m·s/rad]. The barycentre of each link is set to the centre of each link.

In these simulations, we assume that tip of 2nd link and 4th link are always placed $y = 0$ in initial position, that is, when $q_2$ and $q_4$ are given, $q_1$ and $q_3$ are set as $q_1 = -q_2/2.0$ and $q_3 = -(q_2+q_4)/2.0$.

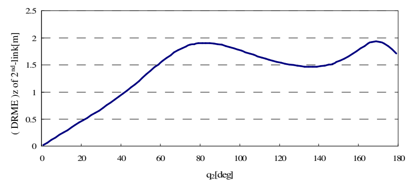


Fig. 7 (DRME)z when $q_4 = 130$[deg]

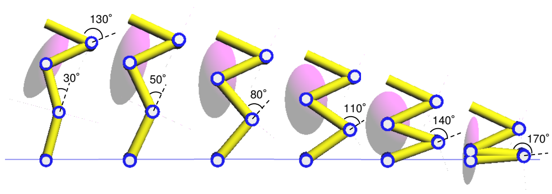


Fig. 8 DRME of various configurations ($q_4 = 130[deg]$)

### 3·1 Simulation of DRMM

In this simulation, we checked DRMM of each configuration of the manipulator when the acceleration of end-link's tip is given, and compared it with Reconfiguraton manipulability Measure(RMM). Fig.5 shows each configuration's distribution of 2nd-link's DRMM and RMM.

From Fig.5, we can find that when $q_2 = 90°$, $q_4 = 90°$, the value of RMM is maximum, and when $q_2 = 118°$, $q_4 = 141°$, the value of DRMM is maximum. The shapes of those time are shown in Fig.4. From Fig.5, we can see the shape of DRMM's peak has a little curved. We think the reason causing this configuration is that we put dynamics into consideration, and the masses of the links have large influence to manipulability.

### 3·2 Verification simulation based on DRM

According to former researches we can know that the condition of manipulator's manipulability can be seen by the volumes of ellipsoids we made. So, in this simulation, we defined the length of DRME in $z$-direction as (DRME)z, and made a task to keep the 3rd and 4th links' configuration and keep them right above the 1st link. Meanwhile, let the tip of 4th link move to find the value of 2nd link's acceleration in $z$-direction.

We keep $q_4 = 130$[deg] and had the relation between $q_2$ and (DRME)z shown in Fig.7. Meanwhile, various configurations of the manipulator in this condition is shown in Fig.8. From z-direction's moving distance of 2nd link showed in Fig.6, we can find that trend showed in Fig.6 is similar to that of Fig.7. According to this result, we can prove the validity and usefulness of DRM.

## 4. Conclusion

In this paper, we proposed a new index named Dynamic Reconfiguration Manipulability(DRM) and showed simulations to indicate the physical properties of DRM, and then proved validity and usefulness of it. For the future work, we plan to add DRM into 7-links manipulators such as PA-10(produced by Mitsubishi Heavy Industries, Ltd.) for trajectory tracking and grinding tasks to improve manipulators' accuracy given to tasks and find the optimum configurations through whole processes. Furthermore, we intend to apply DRM to humanoid robot to analyse humanoid robot's walking states.

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