Temperature Control of a Mold Model using Multiple-input Multiple-output Two Degree-of-freedom Generalized Predictive Control

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***Abstract*—This paper considers an application of two degree-of-freedom generalized predictive control (Two DOF GPC) to an aluminum mold model of temperature control experimental device. In our research, Two DOF GPC can achieve to design the output response for the aluminum plate model independently of modeling error or disturbance. The present study aims at controlling the temperature for various mold models in order to develop the proposed method for application to industry. Therefore, this paper shows an aluminum mold model, which we newly constructed, for temperature control by extending the conventional model. And we increased the number of input and output to examine the interference of heat with temperature control of the derived model by using two DOF GPC. The numerical examples are shown to verify the validity of the model and the proposed method.**

I. Introduction

The present study aims at three dimensional temperature control by expanding the model of one dimensional aluminum plate into two dimensional. This paper uses two degree-of-freedom generalized predictive control because we believe that the concept of prediction is acceptable and reasonable for industry. GPC technique has been first proposed by Clarke and others in 1987 [1]. The control method has features that the objective function includes prediction and control horizons, and control signals are computed by receding their horizons. With these features, the control strategy has been accepted by many of practical engineers and applied widely in industry [2]. Although our method can achieve to design the output response for the aluminum plate model independently of modeling error or disturbance, the previous model was just one dimensional model [3][4][5].

There are a lot of studies about temperature control. For example, the temperature control by GPC has been studied by Zhang and others in 2002, but the control itself deals with PID control by the self-tuning based GPC [7]. Further, the temperature control for the purpose of temperature uniformity by Nanno and others in 2002 has studies gradient of temperature control method by using the PID control [8].
We believe that our study by using Two DOF GPC about temperature control is unique in other ones. We have to consider the application to industrial fields by using this method. But, it could not be said to be sufficient for the application to industry. So this paper aims at three dimensional temperature control by expanding the model of one dimensional aluminum plate into two dimensional one in order to develop thermotherapy machine and to produce products which are made from thermoplastic materials.

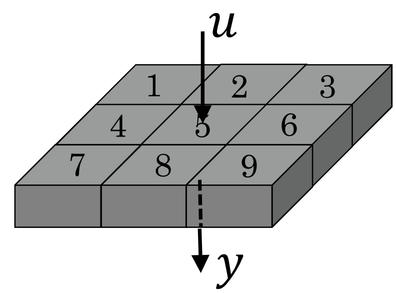

Fig. 1: Aluminum plate model

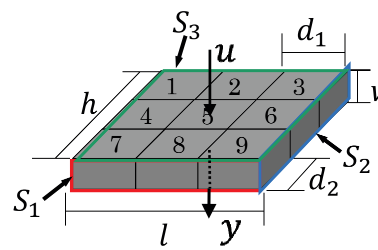

Fig. 2: Parameters of the model

Therefore, this paper shows an aluminum mold model, which we newly constructed, for temperature control by extending the conventional model. And we increased the number of input and output to examine the interference of heat with temperature control of the derived model by using two DOF GPC. The numerical examples are shown to verify the validity of the model and the proposed method.

II. Derivation of Model

Firstly the following model shown in Fig.1 is considered.

The parameters of aluminum plate model are given in Table I.

TABLE I: Parameters for the model

Density of aluminum: $\rho = 2700[\mathrm{kg/m^3}]$
Specific Heat: $c = 0.917[\mathrm{kJ/kgK}]$
Heat transfer coefficient: $k = 20[\mathrm{W/m^2K}]$
Thermal conductivity: $\lambda_f = 238[\mathrm{W/mK}]$
Height and Length: $l = 250[\mathrm{mm}]$
Width: $w = 10[\mathrm{mm}]$

The state variables are defined as the following.

$$x_n = T_n - T_o \tag{1}$$

This model is nine dimensional model with dividing into 3×3 lengthwise and crosswise and $n = 1, 2, \cdots, 9$ to the right from the upper left. Where T_n is temperature of each part of the aluminum plate, T_o is ambient temperature. Then, three laws are used in the derivation of the model. Fourier's law of heat conduction is given by,

$$q = -\lambda_f \frac{d\theta}{dn} \tag{2}$$

Where q $[\mathrm{W/m^2}]$ is heat flow ratio, λ_f $[\mathrm{W/mK}]$ is thermal conductivity, $d\theta/dn$ $[\mathrm{K/m}]$ is temperature gradient. Newton's law of cooling is given by,

$$q = h(\theta_s - \theta_f) \tag{3}$$

h $[\mathrm{W/m^2K}]$ is heat transfer coefficient. The law of heat conduction is given by,

$$dQ = mc \cdot d\theta \tag{4}$$

c $[\mathrm{J/kgK}]$ is specific heat, m $[\mathrm{kg}]$ is mass of each part. Then the following equations of the system are obtained from the state variables of Eq.(1) and the laws of Eq.(2),(3) and (4).

$$\begin{aligned}
mc\frac{dx_1}{dt} &= -(k(\frac{1}{3}S_1 + \frac{1}{3}S_2 + \frac{2}{9}S_3) \\
&\quad +\lambda_f(\frac{S_1}{d_1} + \frac{S_2}{d_2}))x_1 \\
&\quad +(\lambda_f\frac{S_2}{d_2})x_2 + (\lambda_f\frac{S_1}{d_1})x_4 \\
mc\frac{dx_2}{dt} &= -(k(\frac{1}{3}S_1 + \frac{2}{9}S_3) \\
&\quad +\lambda_f(\frac{S_1}{d_1} + 2\frac{S_2}{d_2}))x_2 + (\lambda_f\frac{S_2}{d_2})x_1 \\
&\quad +(\lambda_f\frac{S_2}{d_2})x_3 + (\lambda_f\frac{S_1}{d_1})x_5 \\
mc\frac{dx_3}{dt} &= -(k(\frac{1}{3}S_1 + \frac{1}{3}S_2 + \frac{2}{9}S_3) \\
&\quad +\lambda_f(\frac{S_1}{d_1} + \frac{S_2}{d_2}))x_3 \\
&\quad +(\lambda_f\frac{S_2}{d_2})x_2 + (\lambda_f\frac{S_1}{d_1})x_6 \\
mc\frac{dx_4}{dt} &= -(k(\frac{1}{3}S_2 + \frac{2}{9}S_3) \\
&\quad +\lambda_f(2\frac{S_1}{d_1} + \frac{S_2}{d_2}))x_4 + (\lambda_f\frac{S_1}{d_1})x_1 \\
&\quad +(\lambda_f\frac{S_2}{d_2})x_5 + (\lambda_f\frac{S_1}{d_1})x_7 \\
mc\frac{dx_5}{dt} &= -(k(\frac{2}{9}S_3) + \lambda_f(2\frac{S_1}{d_1} + 2\frac{S_2}{d_2}))x_5 \\
&\quad +u_1 + (\lambda_f\frac{S_1}{d_1})x_2 + (\lambda_f\frac{S_2}{d_2})x_4 \\
&\quad +(\lambda_f\frac{S_2}{d_2})x_6 + (\lambda_f\frac{S_1}{d_1})x_8 \\
mc\frac{dx_6}{dt} &= -(k(\frac{1}{3}S_2 + \frac{2}{9}S_3) + \\
&\quad \lambda_f(2\frac{S_1}{d_1} + \frac{S_2}{d_2}))x_6 + (\lambda_f\frac{S_1}{d_1})x_3 \\
&\quad +(\lambda_f\frac{S_2}{d_2})x_5 + (\lambda_f\frac{S_1}{d_1})x_4 \\
mc\frac{dx_7}{dt} &= -(k(\frac{1}{3}S_1 + \frac{1}{3}S_2 + \frac{2}{9}S_3) \\
&\quad +\lambda_f(\frac{S_1}{d_1} + \frac{S_2}{d_2}))x_7 \\
&\quad +(\lambda_f\frac{S_1}{d_1})x_4 + (\lambda_f\frac{S_2}{d_2})x_8 \\
mc\frac{dx_8}{dt} &= -(k(\frac{1}{3}S_1 + \frac{2}{9}S_3) \\
&\quad +\lambda_f(\frac{S_1}{d_1} + 2\frac{S_2}{d_2}))x_8 + (\lambda_f\frac{S_1}{d_1})x_5 \\
&\quad +(\lambda_f\frac{S_2}{d_2})x_7 + (\lambda_f\frac{S_2}{d_2})x_9 \\
mc\frac{dx_9}{dt} &= -(k(\frac{1}{3}S_1 + \frac{1}{3}S_2 + \frac{2}{9}S_3) \\
&\quad +\lambda_f(\frac{S_1}{d_1} + \frac{S_2}{d_2}))x_9 \\
&\quad +(\lambda_f\frac{S_1}{d_1})x_6 + (\lambda_f\frac{S_2}{d_2})x_8
\end{aligned}$$

From these equations, the state space equations were obtained. For the sake of simple expression of the coefficients for the above equations, each coefficient of the equation for $x_i (i = 1, \cdots, 9)$ is renamed as $a_{ij} (i = 1, \cdots, 9, j = 1, \cdots, 9)$.

$$\begin{aligned}
mc\frac{dx_1}{dt} &= a_{11}x_1 + a_{12}x_2 + a_{14}x_4 \\
mc\frac{dx_2}{dt} &= a_{22}x_2 + a_{21}x_1 + a_{23}x_3 + a_{25}x_5 \\
mc\frac{dx_3}{dt} &= a_{33}x_3 + a_{32}x_2 + a_{36}x_6 \\
mc\frac{dx_4}{dt} &= a_{44}x_4 + a_{41}x_1 + a_{45}x_5 + a_{47}x_7 \\
mc\frac{dx_5}{dt} &= a_{55}x_5 + a_{52}x_2 + a_{54}x_4 + a_{56}x_6 + a_{58}x_8 + u_1 \\
mc\frac{dx_6}{dt} &= a_{66}x_6 + a_{63}x_3 + a_{65}x_5 + a_{69}x_9 \\
mc\frac{dx_7}{dt} &= a_{77}x_7 + a_{74}x_4 + a_{78}x_8 \\
mc\frac{dx_8}{dt} &= a_{88}x_8 + a_{85}x_5 + a_{87}x_7 + a_{89}x_9 \\
mc\frac{dx_9}{dt} &= a_{99}x_9 + a_{96}x_6 + a_{98}x_8
\end{aligned}$$

The result of the above state space equations is given as,

$$\begin{aligned}
\mathbf{x}[k+1] &= \mathbf{A}\mathbf{x}[k] + \mathbf{B}\mathbf{u}[k] \\
\mathbf{y}[k] &= \mathbf{C}\mathbf{x}[k]
\end{aligned}$$

The system matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are defined as follows,

$$\mathbf{A} = \frac{1}{mc}\begin{bmatrix} a_{11} & a_{12} & 0 & a_{14} & & & & & \\ a_{21} & a_{22} & a_{23} & 0 & a_{25} & & & 0 & \\ 0 & a_{32} & a_{33} & 0 & 0 & a_{36} & & & \\ a_{41} & 0 & 0 & a_{44} & a_{45} & 0 & a_{47} & & \\ & a_{52} & 0 & a_{54} & a_{55} & a_{56} & 0 & a_{58} & \\ & & a_{63} & 0 & a_{65} & a_{66} & 0 & 0 & a_{69} \\ & & & a_{74} & 0 & 0 & a_{77} & a_{78} & 0 \\ & 0 & & & a_{85} & 0 & a_{87} & a_{88} & a_{89} \\ & & & & & a_{96} & 0 & a_{98} & a_{99} \end{bmatrix}$$

$$\mathbf{B} = \frac{1}{mc}\begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

$$\mathbf{C} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{x} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 \end{bmatrix}^T$$

Where it assumes that a certain sampling time is considered. Then the state space representation and the derivation of the aluminum plate temperature control model in the nine dimensional has been completed. It is noticed that although we assume that the obtained model is single input single output system and the positions of input and output are $n = 5$ (located in the center part of the model), their positions can be changed easily.

III. Derivation of Controller

A. GPC law

Consider the following m-input m-output system:

$$\mathbf{x}[k+1] = \mathbf{A}\mathbf{x}[k] + \mathbf{B}\mathbf{u}[k] \quad (5)$$
$$\mathbf{y}[k] = \mathbf{C}\mathbf{x}[k] \quad (6)$$

where $\mathbf{x}[k]$, $\mathbf{y}[k]$, $\mathbf{u}[k]$ and k denote state variable, output, input and time step. $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ are constant matrix. The steady state values $\mathbf{x}_\infty$ of $\mathbf{x}[k]$, $\mathbf{u}_\infty$ of $\mathbf{u}[k]$ and $\mathbf{y}_\infty$ of $\mathbf{y}[k]$ are derived as follows.

$$\mathbf{x}_\infty = \mathbf{A}\mathbf{x}_\infty + \mathbf{B}\mathbf{u}_\infty \quad (7)$$
$$\mathbf{y}_\infty = \mathbf{C}\mathbf{x}_\infty \quad (8)$$

Subtracting Eq.(7) and (8) from Eq.(5) and (6).

$$\mathbf{x}[k+1] - \mathbf{x}_\infty = \mathbf{A}(\mathbf{x}[k] - \mathbf{x}_\infty) + \mathbf{B}(\mathbf{u}[k] - \mathbf{u}_\infty)$$
$$\mathbf{y}[k] - \mathbf{y}_\infty = \mathbf{C}(\mathbf{x}[k] - \mathbf{x}_\infty)$$

The following deviation system by defining $\tilde{\mathbf{x}}[k] = \mathbf{x}[k] - \mathbf{x}_\infty$, $\tilde{\mathbf{y}}[k] = \mathbf{y}[k] - \mathbf{y}_\infty$ and $\tilde{\mathbf{u}}[k] = \mathbf{u}[k] - \mathbf{u}_\infty$ is provided.

$$\tilde{\mathbf{x}}[k+1] = \mathbf{A}\tilde{\mathbf{x}}[k] + \mathbf{B}\tilde{u}[k] \quad (9)$$
$$\tilde{\mathbf{y}}[k] = \mathbf{C}\tilde{\mathbf{x}}[k] \quad (10)$$

For this deviation system, the output of j-steps ahead $\tilde{\mathbf{y}}[k+j]$ can be calculated as follows.

$$\tilde{\mathbf{y}}[k+j] = \mathbf{C}\mathbf{A}^j\tilde{\mathbf{x}}[k] + \sum_{i=1}^{j}\mathbf{C}\mathbf{A}^{i-1}\mathbf{B}\tilde{\mathbf{u}}[k+j-i] \quad (11)$$

Since it is assumed that there is no disturbance here, the predicted value of the output of j-steps ahead, $\hat{\tilde{\mathbf{y}}}[k+j|k]$, is equal to Eq.(11). Further, to express a vector form $\hat{\tilde{\mathbf{y}}}[k+j|k]$, the following vectors and matrices are defined.

$$\hat{\tilde{\mathbf{Y}}}[k] = [\hat{\tilde{\mathbf{y}}}^T[k+N_1|k] \cdots \hat{\tilde{\mathbf{y}}}^T[k+N_2|k]]^T$$

$$\tilde{\mathbf{U}}[k] = [\tilde{\mathbf{u}}^T[k] \cdots \tilde{\mathbf{u}}^T[k+N_u-1]]^T$$

$$\mathbf{H} = \begin{bmatrix} \mathbf{C}\mathbf{A}^{N_1} \\ \mathbf{C}\mathbf{A}^{N_1+1} \\ \vdots \\ \mathbf{C}\mathbf{A}^{N_2} \end{bmatrix}$$

$$\mathbf{G} = \begin{bmatrix} \mathbf{C}\mathbf{A}^{N_1-1}\mathbf{B} & \cdots & \mathbf{C}\mathbf{B} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \mathbf{C}\mathbf{A}^{N_u-1}\mathbf{B} & \ddots & \ddots & \ddots & \mathbf{C}\mathbf{B} \\ \vdots & & \vdots & & \vdots \\ \mathbf{C}\mathbf{A}^{N_2-1}\mathbf{B} & \cdots & \cdots & \cdots & \mathbf{C}\mathbf{A}^{N_2-N_u}\mathbf{B} \end{bmatrix}$$

Where $[N_1, N_2]$ and $[1, N_u]$ denote prediction horizon and control horizon. Then the output prediction $\hat{\tilde{\mathbf{Y}}}[k]$ can be expressed as follows.

$$\hat{\tilde{\mathbf{Y}}}[k] = \mathbf{H}\tilde{\mathbf{x}}[k] + \mathbf{G}\tilde{\mathbf{U}}[k] \quad (12)$$

To obtain a control law for realizing the target value tracking, the following performance index J is considered.

$$J = \tilde{\mathbf{Y}}^T[k]\tilde{\mathbf{Y}}[k] + \tilde{\mathbf{U}}^T[k]\mathbf{\Lambda}\tilde{\mathbf{U}}[k] \quad (13)$$

By substituting Eq.(12) into Eq.(13) and having partial differentiation for $\tilde{\mathbf{U}}[k]$, the control law is obtained.

$$\tilde{\mathbf{U}}[k] = -(\mathbf{G}^T\mathbf{G} + \mathbf{\Lambda})^{-1}\mathbf{G}^T\mathbf{H}\tilde{\mathbf{x}}[k]$$

Since $\mathbf{u}[k]$ is the first element of the $\tilde{\mathbf{U}}[k]$, the control input is given as follows.

$$\mathbf{u}[k] = \mathbf{F}_0\mathbf{x}[k] - \mathbf{F}_0\mathbf{x}_\infty + \mathbf{u}_\infty \quad (14)$$

where

$$\mathbf{F}_0 = -[\mathbf{I}_m \ \mathbf{0}_m \ \cdots \ \mathbf{0}_m](\mathbf{G}^T\mathbf{G} + \mathbf{\Lambda})^{-1}\mathbf{G}^T\mathbf{H}$$

Furthermore, assuming steady-state value $\mathbf{y}_\infty$ of the output becomes equal to the target value $\mathbf{r}$, $\mathbf{x}_\infty$ and $\mathbf{u}_\infty$ are given by the following equation.

$$\begin{bmatrix} \mathbf{x}_\infty \\ \mathbf{u}_\infty \end{bmatrix} = \begin{bmatrix} \mathbf{A} - \mathbf{I} & \mathbf{B} \\ \mathbf{C} & \mathbf{0} \end{bmatrix}^{-1}\begin{bmatrix} \mathbf{0} \\ \mathbf{r} \end{bmatrix} \quad (15)$$

From Eq.(15), it can calculate the second and the third term of the right side of Eq.(14) as follows.

$$-\mathbf{F}_0\mathbf{x}_\infty + \mathbf{u}_\infty = -\left\{\mathbf{C}(\mathbf{A} - \mathbf{I} + \mathbf{B}\mathbf{F}_0)^{-1}\mathbf{B}\right\}^{-1}\mathbf{r}$$

Then the control law is expressed by the following equation.

$$\mathbf{u}[k] = \mathbf{F}_0\mathbf{x}[k] + \mathbf{H}_0\mathbf{r} \quad (16)$$

where

$$\mathbf{H}_0 = -\left\{\mathbf{C}(\mathbf{A} - \mathbf{I} + \mathbf{B}\mathbf{F}_0)^{-1}\mathbf{B}\right\}^{-1} \quad (17)$$

B. Two DOF GPC law

The following control law is considered by including integral compensation in order to construct two DOF GPC.

$$\mathbf{u}[k] = \mathbf{F}_0\mathbf{x}[k] + \mathbf{H}_0\mathbf{r} + \mathbf{G}_0\mathbf{z}[k] \tag{18}$$

Where $\mathbf{z}[k]$ is integral compensation, $\mathbf{G}_0$ is integral gain. The control law (Eq.(18)) can make the steady state error zero. However without disturbance and modeling error, it may lead to increase in the control input and delay of tracking the target value in the transient response. So we make the effect of integral compensation appear only when the disturbance and modeling error exist. Also as $\mathbf{e}[k] = \mathbf{r} - \mathbf{y}[k]$, which is the difference between the target value and the output, is used for integral compensation as follows.

$$\mathbf{w}[k] = \frac{1}{\Delta}\mathbf{e}[k] \quad (\Delta = 1 - z^{-1}) \tag{19}$$

In the case that the disturbance or modeling error does not exist, Eq.(18) should be equal to Eq.(17), in order to derive two DOF GPC law. Therefore the following equation is considered by substituting Eq.(17) into Eq.(1), and subtracting $\mathbf{x}[k]$ from both sides.

$$\mathbf{x}[k+1] - \mathbf{x}[k] = (\mathbf{A} - \mathbf{I} + \mathbf{B}\mathbf{F}_0)\mathbf{x}[k] + \mathbf{B}\mathbf{H}_0\mathbf{r}$$

Then

$$\begin{aligned}\mathbf{x}[k] &= (\mathbf{A} - \mathbf{I} + \mathbf{B}\mathbf{F}_0)^{-1}\mathbf{x}[k+1] \\ &\quad -(\mathbf{A} - \mathbf{I} + \mathbf{B}\mathbf{F}_0)^{-1}\mathbf{x}[k] \\ &\quad -(\mathbf{A} - \mathbf{I} + \mathbf{B}\mathbf{F}_0)^{-1}\mathbf{B}\mathbf{H}_0\mathbf{r}\end{aligned} \tag{20}$$

By substituting the above equation into **e**[k] and using Eq.(16), we have.

$$\mathbf{e}[k] = -\mathbf{C}(\mathbf{A} - \mathbf{I} + \mathbf{B}\mathbf{F}_0)^{-1}(\mathbf{x}[k+1] - \mathbf{x}[k])$$

That is, if the disturbance and modeling error do not exist, the integral compensation by the tracking error can be calculated as follows.

$$\begin{aligned}\mathbf{w}'[k] &= \frac{1}{\triangle}\mathbf{e}[k] \\ &= -\mathbf{C}(\mathbf{A} - \mathbf{I} + \mathbf{B}\mathbf{F}_0)^{-1}(\mathbf{A} + \mathbf{B}\mathbf{F}_0) \\ &\quad \cdot(\mathbf{x}[k] - \mathbf{x}[0])\end{aligned} \tag{21}$$

Where $\mathbf{w}'[0]$ is assumed to be 0. Since Eq.(21) represents the integral compensation when the disturbance does not exist, the integral compensation $\mathbf{z}[k]$ of two DOF GPC is given by the following equation.

$$\mathbf{z}[k] = \mathbf{w}[k] - \mathbf{w}'[k]$$

From this equation, it always becomes $\mathbf{z}[k] = \mathbf{0}$ if the disturbance and modeling error does not exist, that is, the effect of the integral compensation does not appear. Thus, the control law of two DOF GPC is given.

IV. Simulation

A. Application of the Control Law

On the model obtained in the section II, the control systems for the three cases are applied. Control target of temperature is to increase $4K$ from ambient temperature. In addition, the disturbanse temperature $0.5K$ is added at $t = 5000$[s]. The

TABLE II: Parameters for GPC

$N_1 = 10$
$N_2 = 120$
$N_u = 2$
$\mathbf{\Lambda} = 0.01\mathbf{I}$
$\mathbf{G}_0 = 0.2\mathbf{I}$

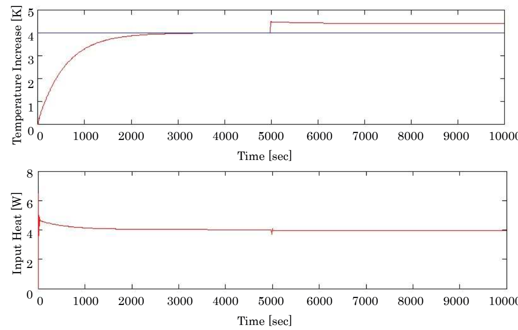

Fig. 3: Simulation result by GPC with no integration (Eq.(16))

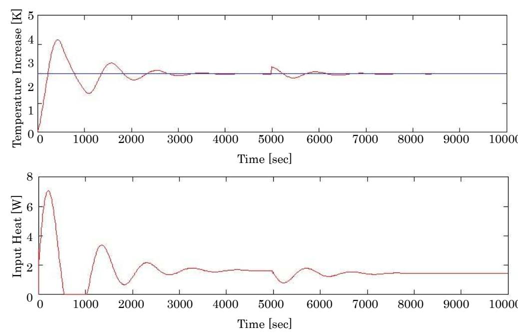

Fig. 4: Simulation result by GPC with integration ($\mathbf{z}[k] = \mathbf{w}[k]$)

design parameters of GPC are given in Table II. The above graph is the simulation result when applying GPC. In Fig.3, we can confirm that the output follows the target value in the steady state. However, since the disturbance of stepwise is added , the output is no longer follow the target value by the influence of the disturbance. Fig.4 shows the simulation result by GPC including the integrator. In this case, we can confirm that the output follows the desired value by canceling the influence of the disturbance, but it would cause an overshoot. In Fig.5, it composes to the control system by two DOF GPC.

In this case, overshoot can be suppressed, and the output can follow the target value by composing two DOF GPC.

B. Confirmation of the Model

In order to check the validity of the model, the step response for the model which is determined in section II, is checked. This case confirms whether the heat spread is natural or not. From Fig.7 to Fig.12, the temperature increase and their distributions are shown in the case that the heat input is applied at each part. At this time, the amount of heat input is always constant and are applying with 15[W]. Further, the output when the inputs to n_1, n_3, n_7 and n_9 is confirmed by Fig.7. When the inputs to n_2, n_4, n_6 and n_8 were given, the

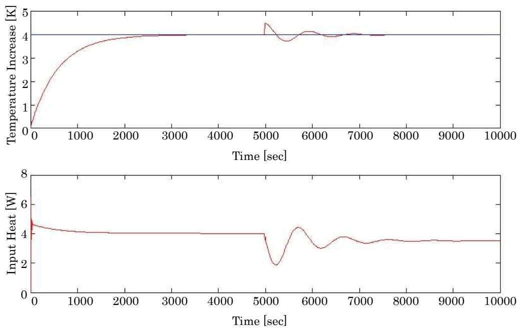

Fig. 5: Simulation result by two DOF GPC($\mathbf{z}[k] = \mathbf{w}[k] - \mathbf{w'}[k]$)

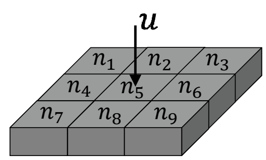

Fig. 6: The Simulation Model of Aluminum

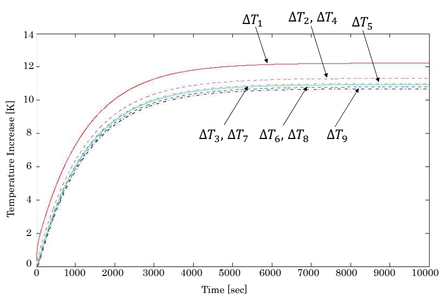

Fig. 7: Temperature increase(input to n_1, n_3, n_7 and n_9)

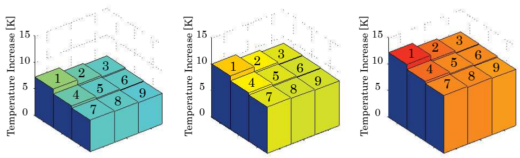

Fig. 8: Temperature distribution (input to n_1)

result is shown in Fig.9. Moreover, when the input to n_5 was given, the result is shown in Fig.11. Also, Fig.8, 10 and 12 show the temperature distribution of each part (in the case for the input to n_1, n_2 or n_5) at $t = 500$[s], 1000[s] and 4500[s].

It found that the heat spreads concentrically from the input point based on the results in Fig.7, 9 and 11 and the results in Fig.8, 10 and 12. Therefore, it can find that this model is appropriate to consider the practical case.

C. Mold Model

17-dimensional mold model is constructed in the same method to describe in Section.2. The model shown in Fig.13.

The material is aluminum, the model parameters are given

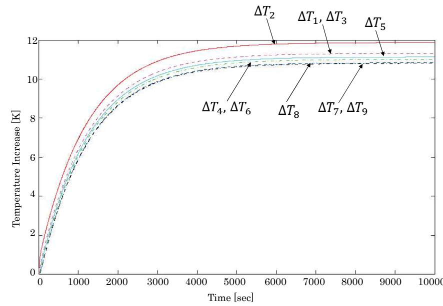

Fig. 9: Temperature increase(input to n_2, n_4, n_6 and n_8)

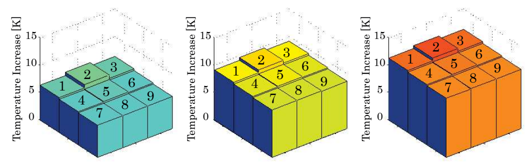

Fig. 10: Temperature Distribution(input to n_2)

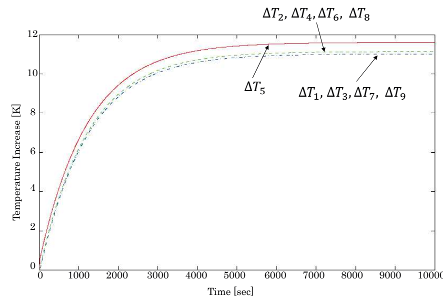

Fig. 11: Temperature increase(input to n_5)

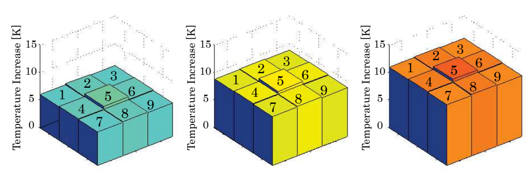

Fig. 12: Temperature distribution(input to n_5)

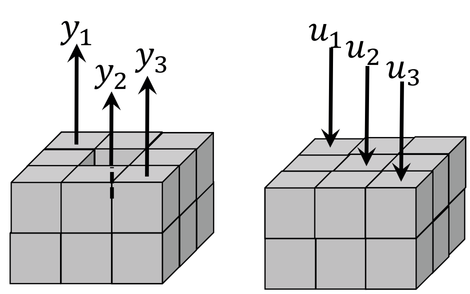

Fig. 13: The mold model (Left:Top Right:Bottom)

in Table I except for hight and length $l = 240[mm]$. First, Fig.14 shows the simulation result of the two DOF GPC in single-input single-output system through the mold model shown in Fig.13. The input is given to u_2 and the output is

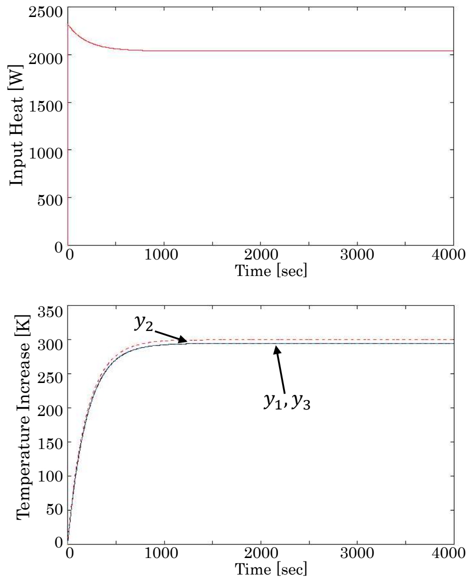

Fig. 14: Simulation result for the model of single-input single-output system

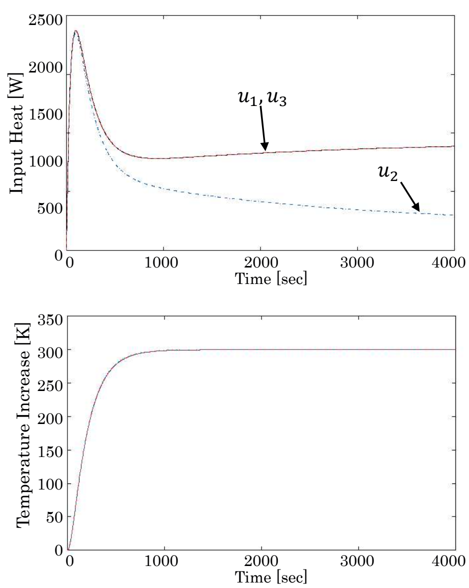

Fig. 15: Simulation result for the model of multi-input multi-output system

measured from y_2. The target temperature is set to $300K$ which is a target value from initial temperature when performing the molding of thermosetting material in the practical mold.

From this result, it can be confirmed that a single-input single-output system does not match the target value for y_1 and y_3 because the controlled value is just y_2 only. Further changed from single-input single-output system to multi-input multi-output system, two DOF GPC is executed for the mold model. Fig.15 shows the simulation result.

From this result, it can confirm that each input would change and interfere with each other, compared with the case of single-input single-output system. From the above result, it can find the effectiveness of two DOF GPC for the multi-input multi-output system. Also, as compared with the single-input single-output system, it can find the effectiveness of the multi-input multi-output system in the meaning of reduction of the amount of heat input.

V. Conclusion

This paper extended the aluminum plate temperature control experimental device model to aluminum mold model through the previous study. And the control system for the model was constructed and the simulation result was given to verify the validity of the mold model and the effectiveness of two DOF GPC. As future works, it is necessary to confirm consistency between the simulation model and the experimental device. Also, it is necessary to confirm the validity of the proposed model by using the experimental device.

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